SFB 649 Discussion Paper 2014-019

Unemployment benefits extensions at the zero lower bound on nominal interest rate

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This research was supported by the Deutsche Forschungsgemeinschaft through the SFB 649 "Economic Risk".

http://sfb649.wiwi.hu-berlin.de
ISSN 1860-5664

SFB 649, Humboldt-Universität zu Berlin
Spandauer Straße 1, D-10178 Berlin

SFB 649 ECONOMIC RISK BERLIN

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February 10, 2014

Abstract

In this paper we investigate the impact of the recent US unemployment benefits extension on the labor market dynamic when the nominal interest rate is held at the zero lower bound (ZLB). Using a New Keynesian model, our quantitative experiments suggest that, in contrast to the existing literature that ignores the liquidity trap situation, unemployment benefits expansions cause a wage and inflationary pressure which curb the increase in real interest rate and slightly reduce unemployment at the ZLB. Outside the ZLB, it has adverse effects, meaning that unemployment insurance benefits should be adjusted according to the macroeconomic conditions if the scope is to reduce unemployment. Furthermore, the ZLB amplifies the labor market downturn. An unconstrained monetary policy rule, i.e. negative interest rate, could have reduced the unemployment rate by around 1 percentage point in the trough of the recession.

Keywords: Zero lower bound, New Keynesian models, Matching frictions, Monetary policy, Unemployment benefits extensions.

JEL Classification: E24, E31, E32, E43, E52, E62

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We thank seminars and conferences participants (the SaM workshop, T2M, the Schumpeter seminar in Humboldt Berlin, the ETEPP Winter School, the doctoral workshop in macroeconomics of Konstanz and the EPEE seminar in Evry) for their helpful comments on an earlier version of this paper. We are also grateful to Xavier Fairise, François Langot, Stéphane Moyen, Ferhat Mihoubi, Thepthida Sopraseuth and DOLETA (Stapleton Kevin

1 Introduction

Against the dramatic increase of the US unemployment rate following the Great Recession, policy makers have triggered, among various fiscal interventions, an unprecedent unemployment insurance (UI) benefits extension. The benefit duration increased up to 99 weeks from the standard 26 weeks. This policy has been frequently pointed out as an important factor in exacerbating unemployment because it would have reduced job search and job acceptance. However, most of the discussion has left aside an important aspect of the economic environment in which the policy has been conducted: the liquidity trap. In a liquidity trap, the monetary policy is no longer able to provide the appropriate stimulus since interest rates slash the ZLB. The output decline calls for negative real interest rate. But low inflation or even deflation makes the real interest rate too high which deepens the recession (Krugman (1998), Benhabib et al. (2002), Fernández-Villaverde et al. (2012)) and makes employers less prone to invest in job creation. As mentioned by Hall (2011) and Hall (2013), high discounts would be likely to imply high unemployment. In this line of research, Schmitt-Grohé & Uribe (2012) have shown that the ZLB combined with downward nominal wage rigidity can lead to a jobless recovery. Output growth recovered but unemployment lingered because the deflationary spiral induced by the liquidity trap makes real wages too high to be compatible with full employment.

Some questions naturally arise: how much of the observe increase in unemployment is due to the ZLB and to the benefits extension? Do extended benefits have different effects in a liquidity trap? How well does the UI extension push wages and inflation upwards? In a demand-driven recession characterized by high discounts, the impact of a fiscal intervention hinges on its ability to create an inflationary pressure. The originality of this paper is to investigate how the UI extension shapes the unemployment rate during the 2008 crisis when the nominal interest rate is held at the ZLB and when the inflationary effects of the policy are taking into account. For this purpose we use a New Keynesian business cycle model with search and matching frictions and price rigidities. We exploit aggregate data from unemployment insurance to quantify the stimulus plan and perform counterfactual experiments to identify the effects of unemployment benefits.

The UI extension effect has been widely criticized on behalf of the standard disincentive effects on job search decisions. Barro (2010), for instance, argues that it subsidized unemployment and led to insufficient job search and job acceptance. According to his own calculations *the jobless rate could be as low as*

and Kim Hyunchung) for their helpful advice. Finally, we thank Maud Bossard and Lusine Nazaretyan for the rereading of the article. This research was supported by the Deutsche Forschungsgemeinschaft through the SFB 649 "Economic Risk". The usual disclaimer applies.

6.8%, instead of 9.5% if jobless benefits hadn't been extended to 99 weeks. Empirical studies found however, a much more modest value. Rothstein (2011) shows that UI extensions raised the unemployment rate in early 2011 by only about 0.1 to 0.5 percentage point, Fujita (2011) by about 0.8 to 1.8 and Farber & Valletta (2013) by 0.4. Interestingly, Hagedorn et al. (2013) show that the response of the job finding rate mainly explain the persistent increase in unemployment while the search intensity plays little. They argue that the wage pressure induced by the benefits extension would have reduced the incentive for firms to invest in job creation. In a general equilibrium framework, Nakajima (2012) found that the 2009 UI extension has raised the unemployment rate by 1.4 percentage points. On the opposite, Krugman (2013) argues that slashing unemployment benefits - which would have the side effect of reducing incomes and hence consumer spending - would not create more jobs but just make the situation worse because employment is limited by demand, not supply.

If, as mentioned Hagedorn et al. (2013), the UI extension has increased wages, the overall impact is ambiguous due to the inflationary pressure that may curb the increase in the real interest rate. In the literature presented above, the study of unemployment benefits abstracts from the liquidity trap situation and macro models assumes that the recession is driven by productivity shocks. This is at odds with the data since negative supply shocks cause an inflationary pressure and can not affect the interest rate in a way that they mimic the financial market turmoils. We argue that the outcome of the UI extension is strongly linked to the response of the real interest rate. The assessment of unemployment benefits must embed the nominal sphere, a proper demand-driven recession and must consider the macroeconomic conditions.

We show that when the economy enters in a liquidity trap the unemployment rate can dramatically increase. While an increase in unemployment benefits always raises the unemployment rate in normal time, it may have opposite effects in a liquidity trap, depending on the shock that sent the economy to the ZLB. The counterfactual exercise shows that the unemployment benefits extension triggered following the Great Recession would have slightly reduced unemployment but only when the nominal interest rate has reached the ZLB. We also use alternative specifications of unemployment benefits and alternative calibrations of search intensity, labor market frictions and real wage rigidities. It is shown that the UI extension, even in the worst case considered, does not cause a sizeable increase in unemployment as long as the interest rate is held at the ZLB.

The rest of the paper is organized as follow. Section 2 is devoted to the presentation of the New Keynesian DSGE model. Section 3 addresses calibration. Simulations and counterfactual experiments are presented in Section 4. Section 5 concludes. We provide a separate appendix describing unemployment benefits multiplier, the model, the calibration and the solution method.

2 The model

We use a baseline New Keynesian DSGE model with search and matching frictions (Mortensen & Pissarides (1994)). The model is characterized by nominal price rigidities (Rotemberg's style), monopolistic competition, and a feedback Taylor rule for monetary policy. We focus on the flow of workers between employment and unemployment. Time is discrete and our economy is populated by homogeneous workers and firms. Producing firms are large and employ many workers as their only input into the production process. Labor may be adjusted through the extensive margin (employment), individual hours are fixed. Wages are the outcome of a bilateral Nash bargaining process between each large firm and each worker. We calibrate the model on US data. Aggregate shocks to the discount factor fuel up the cycle.

2.1 The labor market

The search process and recruiting activities are costly and time-consuming for firms and workers. A job may either be filled and productive, or unfilled and unproductive. To fill their vacant jobs, firms publish adverts and screen workers, incurring hiring expenditures. Workers are identical and they may either be employed or unemployed. The number of matches, m_t, is given by the following CES matching function[1]:

$$m_t = \left((e_t s_t)^{-\gamma} + v_t^{-\gamma} \right)^{-\frac{1}{\gamma}} \leq \min(e_t s_t, v_t) \tag{1}$$

where $v_t \geq 0$ denotes the mass of vacancies, $s_t \geq 0$ represents the mass of searching workers and $e_t \geq 0$ stands for the endogenous search effort. The labor force, L, is assumed to be constant over time. Assuming $L = 1$ allows us to treat aggregate labor market variables in number and rate without distinction. The matching function (1) is increasing and concave in its two arguments. A vacancy is filled with probability $q_t = m_t / v_t$ and the job finding probability per efficiency units of worker search is $f_t = m_t / (e_t s_t)$.

2.2 The sequence of events

Following Hall (2005), we abstract from job destruction decisions by assuming that in each period a fixed proportion of existing jobs are exogenously destroyed at rate ρ^x. n_t denotes employment in period t. It has two components:

[1]The use of a CES matching function, instead of the standard Cobb-Douglas one allows to evaluate how a change in the degree of matching frictions affects our results. The CES encompasses the Cobb-Douglas case. An increase in γ reduces the mismatches and frictions. If $\gamma \to +\infty, m_t = \min(s_t, v_t)$ which is a frictionless hiring process. Furthermore, it ensures the job finding and filling probabilities remains below 1.

new and old workers. New employment relationships are formed through the matching process in period t. The number of job seekers is given by:

$$s_t = 1 - (1 - \rho^x) n_{t-1} \tag{2}$$

This definition has two major consequences. First, it allows workers who lose their job in period t to have a probability of being employed in the same period. Second, it allows the model to make a distinction between job seekers and unemployed workers $u_t = 1 - n_t$. The latter receives unemployment benefits. The employment law of motion is given by:

$$n_t = (1 - \rho^x) n_{t-1} + m_t \tag{3}$$

2.3 The representative household

There is a continuum of identical households of measure one indexed by $i \in [0, 1]$. Each household may be viewed as a large family. There is a perfect risk sharing, family members pool their incomes (labor incomes and unemployment benefits) that are equally redistributed. We suppose that households have preference over different consumption varieties. Good varieties are indexed by $j \in [0, 1]$. Each household maximizes the aggregate consumption using a Dixit-Stiglitz aggregator of differentiated goods c_{jt} and faces the following demand function[2]:

$$c_{jt} = \left(\frac{p_{jt}}{p_t} \right)^{-\epsilon} c_t \tag{4}$$

which describes the optimal level of c_{jt} and where c_t is aggregate consumption. The nominal price index is defined by $p_t = \left[\int_0^1 p_{jt}^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}}$. The second problem that households solve is the maximization of aggregate consumption c_t:

$$\max_{\Omega_t^H} E_0 \sum_{t=0}^{\infty} \left(\prod_{k=0}^{t} \beta_k \right) \left[\frac{{c_t}^{1-\sigma}}{1-\sigma} - \ell \frac{{n_t}^{1+\phi}}{1+\phi} \right] \tag{5}$$

n_t is the level of employment supplied by households. The parameters $\sigma > 0$ and $1 + \phi > 0$ denote the coefficient of risk aversion and the inverse of the Frisch elasticity respectively. β_t represents a discount factor shock. The representative household chooses the set of processes $\Omega_t^H = \{c_t, e_t, d_t, n_t\}_{t=0}^{\infty}$ taking

[2]We skip intermediary equations since they are standard.

as given the set of processes $\{p_t, w_t, i_t, f_t\}_{t=0}^{\infty}$, and the initial wealth (d_0) so as to maximize their utility subject to the budget constraint:

$$p_t s_t k(e_t) + p_t c_t + d_t = d_{t-1}(1 + i_{t-1}) \quad + \quad w_t n_t + (1 - n_t)\, b_t + \Pi_t + T_t \quad (6)$$

and the law of motion of employment:

$$n_t \quad = \quad (1 - \rho^x) n_{t-1} + f_t s_t e_t \quad (7)$$

$k(e_t)$ is the cost of searching a job for a job seeker. d_t is the household's holding of one period domestic bonds at date t. The corresponding nominal interest rate is i_t. w_t is the nominal wage level. Π_t represents profits from holding shares in domestic goods firms. T_t is a lump-sum tax and b_t denotes unemployment benefits. The optimality conditions of the household's problem are:

$$\begin{aligned}
\varphi_t &= \lambda_t \left(w_t^R - b_t^R \right) - \ell n_t^{\phi} && (8)\\
&\quad + E_t \beta_{t+1} (1 - \rho^x) \left(k(e_{t+1}) \lambda_{t+1} + (1 - f_{t+1} e_{t+1}) \varphi_{t+1} \right) && (9)\\
\lambda_t &= c_t^{-\sigma} && (10)\\
\lambda_t &= (1 + i_t) E_t \beta_{t+1} \lambda_{t+1} \frac{p_t}{p_{t+1}} && (11)\\
k'(e_t) &= \varphi_t \frac{f_t}{\lambda_t} && (12)
\end{aligned}$$

Equation (9) is the marginal value of employment for a worker where $w_t^R = w_t / p_t$ and $b_t^R = b_t / p_t$ denotes the real wage and the real unemployment benefits level respectively. λ_t is the Lagrange multiplier on the budget constraint. Equation (11) defines the standard Euler equation and (12) stands for the optimal searching strategy.

2.4 Firms

There is a continuum of producers in a monopolistically competitive market indexed by j. They use labor as their only input and sell output to the representative household. They face quadratic price adjustment cost (Rotemberg-style). The production function of a firm j using a fraction n_{jt} of total employment such that $\int_0^1 n_{jt} dj = n_t$ is given by:

$$y_{jt} = n_{jt}^{\alpha} \quad (13)$$

α is the employment share of production in the consumption good. The optimization problem of the firm j consists in choosing the set of processes $\Omega_{jt}^F =$

$\{v_{jt}, p_{jt}, n_t\}_{t=0}^{\infty}$ taking as given the set of processes $\{p_t, w_{jt}, q_t\}_{t=0}^{\infty}$. Each j producer maximizes the following intertemporal function:

$$\max_{\Omega_t^P} E_0 \sum_{t=0}^{\infty} \left(\prod_{k=0}^{t} \beta_k \right) \frac{\lambda_t}{\lambda_0} \Pi_{jt} \tag{14}$$

$$\text{where } \Pi_{jt} = \left[\frac{p_{jt}}{p_t} y_{jt} - \frac{w_{jt}}{p_t} n_{jt} - \kappa v_{jt} + y_t \Gamma^{\pi}(p_{jt}) \right]$$

subject to the production function (13) and the following evolution of employment:

$$n_{jt} = (1 - \rho^x) n_{jt-1} + q_t v_{jt} \tag{15}$$

$\frac{p_{jt}}{p_t}$ is the relative price which coincides with the marginal cost of the producing firm. As in Rotemberg, adjusting prices incurs a cost :

$$\Gamma^{\pi}(p_{jt}) = \frac{\psi}{2} \left(\frac{p_{jt}}{\pi p_{jt-1}} - 1 \right)^2 \tag{16}$$

This cost is assumed to be proportional to the output level y_t. Inflation is defined as the gross inflation rate $\pi_t = p_t / p_{t-1}$. ψ is the price adjustment cost parameter and $\bar{\pi}$ is the steady state inflation. Hiring is costly and incurs a cost κ per vacancy posted (with $\int_0^1 v_{jt} dj = v_t$). It is paid by the firm as long as the job remains unfilled. Since all firms choose the same price and the same number of vacancies in equilibrium we can drop the index j by symmetry. The optimality conditions of the above problem are:

$$q_t \mu_t = \kappa \tag{17}$$

$$\mu_t = mc_t \alpha \frac{y_t}{n_t} - w_t^R + (1 - \rho^x) E_t \beta_{t+1} \frac{\lambda_{t+1}}{\lambda_t} \mu_{t+1} \tag{18}$$

$$\begin{aligned} 0 &= (1 - \epsilon) + \epsilon mc_t - \psi \frac{\pi_t}{\pi} \left(\frac{\pi_t}{\pi} - 1 \right) \\ &+ E_t \beta_{t+1} \frac{\lambda_{t+1}}{\lambda_t} \psi \frac{\pi_{t+1}}{\pi} \left(\frac{\pi_{t+1}}{\pi} - 1 \right) \frac{y_{t+1}}{y_t} \end{aligned} \tag{19}$$

where μ_t is the Lagrangian multiplier associated with the employment evolution that gives the expected marginal value of a job for the firm[3]. mc_t is the Lagrange multiplier associated with the individual consumption demand. Combining the two first-order conditions (17) and (18) gives the job creation condition:

$$\frac{\kappa}{q_t} = mc_t \alpha \frac{y_t}{n_t} - w_t^R + (1 - \rho^x) E_t \beta_{t+1} \frac{\lambda_{t+1}}{\lambda_t} \frac{\kappa}{q_{t+1}} \tag{20}$$

[3]It is obtained by deriving the program (14) with respect to n_t.

This condition shows that the expected gain from hiring a new worker is equal to the average cost of search (which is the marginal cost of a vacancy times the average duration of a vacancy $1/q_t$).

2.4.1 Wage setting

At equilibrium, filled jobs generate a return (the firm marginal value of the job μ_t plus the worker marginal value of the job φ_t) greater than the value of a vacant job and of an unemployed worker. Nominal wages are determined through an individual Nash bargaining process between each worker and his employer who share the total surplus of the match. The outcome of the bargaining process is given by the solution of the following maximization problem:

$$\max_{w_t} \quad \left(\frac{\varphi_t}{\lambda_t}\right)^{1-\xi} \mu_t^{\xi} \tag{21}$$

The optimality condition of the above problems is given by:

$$\xi \frac{\varphi_t}{\lambda_t} \quad = \quad (1-\xi)\mu_t \tag{22}$$

where $\xi \in [0,1]$ and $1-\xi$ denote the firms and workers bargaining power respectively. Using the definition of μ_t and φ_t, we have

$$\begin{aligned} w_t^R = (1-\xi)&\left(mc_t \alpha \frac{y_t}{n_t} + E_t \beta_{t+1} \frac{\lambda_{t+1}}{\lambda_t}(1-\rho^x)\kappa\theta_{t+1}e_{t+1}\right) \\ &+\xi\left(b_t^R + \frac{\ell n_t^{\phi}}{\lambda_t} - E_t\beta_{t+1}(1-\rho^x)\frac{\lambda_{t+1}}{\lambda_t}k(e_{t+1})\right) \end{aligned} \tag{23}$$

As it is standard in matching models, the real wage is a weighted sum of the worker's outside option and their contribution to the product.

2.5 The monetary and fiscal authorities

We assume that the central bank adjusts the nominal interest rate i_t in response to deviations of inflation and output from their steady-state value according to a Taylor-type rule:

$$1+i_t = \begin{cases} (1+i_{t-1})^{\rho_i}\left(\frac{\pi}{\beta}\left(\frac{\pi_t}{\pi}\right)^{\rho_\pi}\left(\frac{y_t}{y}\right)^{\rho_y}\right)^{1-\rho_i} & \text{if} \quad i_t > 0 \\ 1 & \text{otherwise} \end{cases} \tag{24}$$

We assume that unemployment benefits are proportional to the aggregate wage and obey to the following rule:

$$b_t^R = \tau_t w_t \tag{25}$$

where τ_t is the replacement rate. It follows an AR(1) process. The fiscal authority finances unemployment benefits b_t through the lump-sum tax T_t. Formally the fiscal budget rule satisfies:

$$d_t + b_t(1 - n_t) = (1 + i_{t-1})d_{t-1} + T_t \tag{26}$$

2.6 Market clearing

The aggregation of individual profits Π_t is given by:

$$\Pi_t = p_t y_t - n_t w_t - p_t y_t \Gamma_t^{\pi} \tag{27}$$

Equation (26) together with the budget constraint (6) and the profit (27) give the aggregate resource constraint:

$$y_t \left[1 - \frac{\psi}{2} \left(\frac{\pi_t}{\pi} - 1 \right)^2 \right] = c_t + \kappa v_t + s_t k(e_t) \tag{28}$$

3 Model calibration

Quarterly frequencies are assumed in our calibration. The benchmark calibration is standard and follows Christiano et al. (2011), Fernández-Villaverde et al. (2012), Aruoba & Schorfheide (2013). The model is solved using a Parameterized Expectation Algorithm (PEA) with regime switching[4].

Preferences, production and shocks: We set the steady state discount factor to 0.996 and the inverse of the Frisch elasticity to 1. The risk aversion coefficient σ is set to 1 and, following the standard approach, the elasticity of substitution between goods $\varepsilon = 6$, which gives a gross markup of about 1.2. The elasticity of output with respect to employment (α) is equal to 1 in our benchmark. β_t follows an AR(1) process: $\log \beta_t = \rho_\beta \log \beta_{t-1} + (1 - \rho_\beta) \log \beta + \sigma_\beta \varepsilon_t^\beta$ where $\rho_\beta = 0.85$ and $\sigma_\beta = 0.002$ in order to match the average time spent at the ZLB of 5%.

[4] The ZLB can not been accurately studied when the model is solved using linear-approximation methods, see Braun et al. (2012) for comparison. A full description of the algorithm is provided in the separate appendix.

Variables	Symbol	Value	Source
Discount factor	β	0.996	Standard
Risk av. coefficient	σ	1.00	Standard
Elast. of subst. between goods	ϵ	6.00	Standard
Frisch elasticity	ϕ	1.00	Standard
Annual steady state inflation	π	1.005	Target
Production function elasticity	α	1.00	Standard
Autocorr. coefficient	ρ_β	0.85	FGGR
Std. of β_t	σ_β	0.002	Target 5pct at the ZLB
Vacancy posting costs	κ	0.05	Target $\kappa v \simeq 0.01y$
Search cost curvature	η	2	Assumed
Disutility of search	k_0	0.02	Deduced
Disutility of labor	ℓ	0.77	Deduced
Replacement rate	ρ^R	0.15	See appendix
Matching frictions	γ	2.74	Target $q \equiv 0.9$
Worker bargaining power	ξ	0.50	PP
Price adjustment	ψ	90.00	$\simeq$ Calvo 0.75
Response to inflation	ρ_π	1.50	Standard
Response to output	ρ_y	0.25	Standard
Interest rate smoothing	ρ_i	0.60	Target ZLB 2009Q1
Autocorr. coefficient b_t	ρ_b	0.80	See appendix
Std. of b_t	σ_b	0.0025	See appendix

Table 1: **Parameters for the benchmark** *PP:Pissarides & Petrongolo (2001), FGGR: Fernández-Villaverde et al. (2012).*

Labor market, stocks and flows: The US unemployment rate u is 5.5% on average over several decades. We set the probability of being unemployed $\rho^x = 10.61\%$ as in the data. At the steady state, the number of matches must be equal to the number of separations: $m = \rho^x n$ with $n = 1 - u = 94.5$. We get the number of job seekers from the definition $s = 1 - (1 - \rho^x)n$. Setting $e = 1$, involves a job finding rate $f = m/(es) = 64.6\%$ and a hiring rate $m/n = 10.61\%$, close to the data (10.65%) from the BLS. Following Andolfatto (1996), the rate at which a firm fills a vacancy is about 0.9. From the CES matching function, γ is calculated in such a way that $q = 0.9$. It follows that $v = m/q$. According to Pissarides & Petrongolo (2001) $\xi = 0.5$. The costs of posting vacancies κ is about less than 1% of the GDP. Following Merz (1995), the cost of search takes the form: $k(e) = k_0 e^\eta$ with $k_0 > 0$ and $\eta > 1$. We first assume that $\eta = 2$. We come back latter on this assumption. The remaining parameter ℓ and k_0 are set to balance the steady state real wage equation (23) and the optimal search strategy equation (12).

Monetary and fiscal policy: Inflation at the steady state is about 2% on the an-

nual basis on average. With β being equal to 0.996, the Euler equation involves an annual steady state nominal interest rate of 3.6% and a real interest rate of about 1.6%. Prices adjust infrequently. We assume that $\psi = 90$ which corresponds to a Calvo parameter of 0.75 when $\varepsilon = 6$ in the log-linearized Phillips Curve, as it is standard. From popular DSGE estimations, we have $\rho_\pi = 1.5$, and $\rho_y = 0.25$. ρ_i is set to match the path of the nominal interest rate over 1998Q4-2013Q2, especially the period of ZLB. We found that $\rho_i = 0.6$ which is less than conventional values found in estimations but still consistent with the rapid decline of the interest rate in 2008. Finally, we assume that the replacement ratio τ_t follows an AR(1) process: $\log \tau_t = \rho_\tau \log \tau_{t-1} + (1 - \rho_\tau) \log \tau + \sigma_\tau \varepsilon_t^\tau$. We adopt a modified version of the OECD methodology to calculate the steady state replacement rate. It is found to be equal to 0.15. The volatility and the persistence of the AR(1) are determined according to the time series we built (see appendix for details). The calibration is summarized in Table 1.

4 Quantitative evaluation of the model

4.1 How does the ZLB impact the economy?

We first present some intuitive results on the ZLB. To understand how the ZLB impacts the labor market, we compute the response of the variables following a sufficiently large demand shock to send the economy on the ZLB (see Figure 1). The size of the shock is calibrated to match the observed increase in unemployment. It reached 10% in October 2009. In our model such a shock involves a fall in output of 5% and a fall in the quarterly gross inflation rate of about 1.5%, broadly consistent with the data. The implied ZLB spell is equal to six quarters. Firms cut vacancies by around 45% which is a similar decrease in the data between 2007Q4 and 2009Q4. The real interest rate increases on impact and gently returns to its equilibrium value.

The red dotted line shows the response of the variables if the nominal interest rate is not constrained by the ZLB. It is shown that the ZLB amplifies the propagation of the labor market downturn. Without any binding constraint, the nominal interest rate falls to -2.5% which implies a decrease in the real interest rate and limits the increase in unemployment. The initial fall in vacancies and in the hiring rate are 10% lower. The faster stabilization in real wage dampens the decrease in the marginal cost and the deflationary pressure. Search intensity falls less than in the constrained case. The drop in output is found to be larger (about 1% point) on impact and the subsequent periods when the economy is constrained by the ZLB.

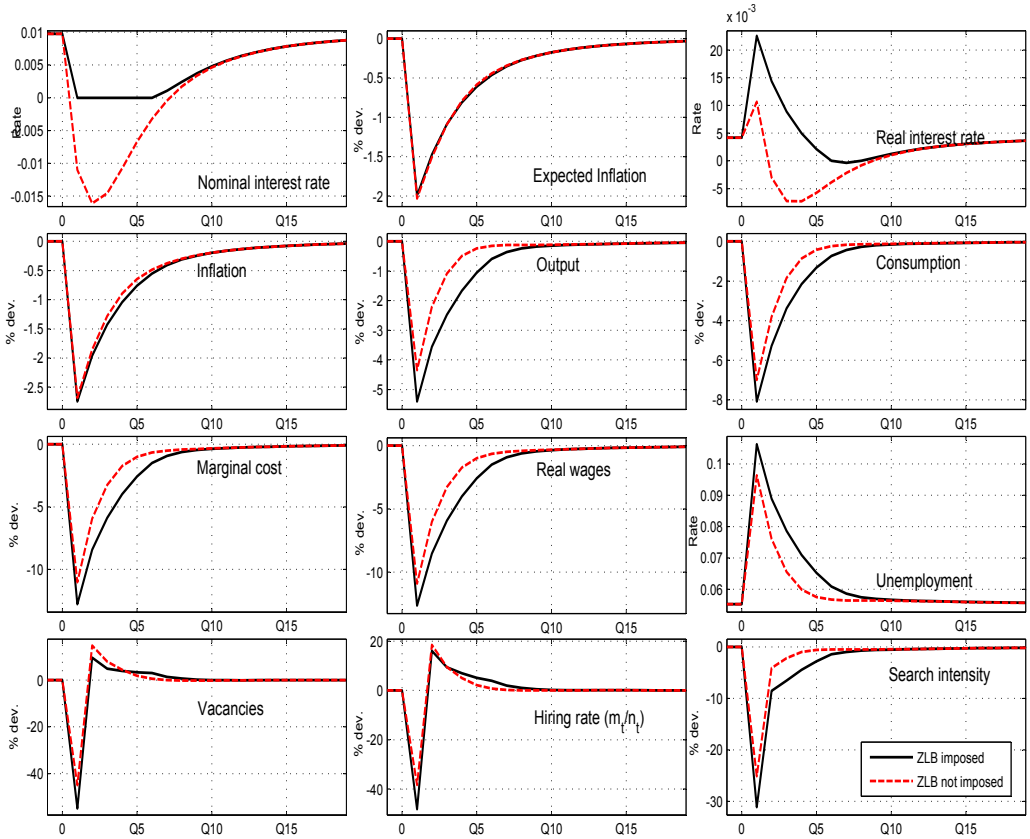

Figure 1: ***Impulse response function - counterfactual analysis.***

4.2 Unemployment benefits multipliers

Before running the counterfactual experiments we show preliminary results on the fiscal multiplier resulting from a one-period and temporary increase in the replacement rate. Figure 2 shows two types of multiplier[5]: the marginal multiplier and the cumulative multiplier. The first one measures the marginal output gain from an increase in unemployment benefits and the second discounts and sums up to date the whole output gain since unemployment benefits have increased.

In the standard DMP model, an increase in the unemployment benefits raises the workers' outside option, which is the present value of being unemployed. It strengthens workers' bargaining position and allows them to claim for higher wages. In addition, the decrease in workers' search intensity reduces firms bargaining position because it makes the outside option more valuable. Firms become less prone to hire workers and cut the number of vacant jobs. The labor market tightness falls and the unemployment rate increases. This is the basic story in normal circumstances. In this case the mul-

[5]See the separate appendix for details.

tiplier (solid black line) jumps down. One additional unit of unemployment benefits causes a 0.2 point decline in output[6] on impact and involves a negative cumulative multiplier of about 0.3 in the long-run.

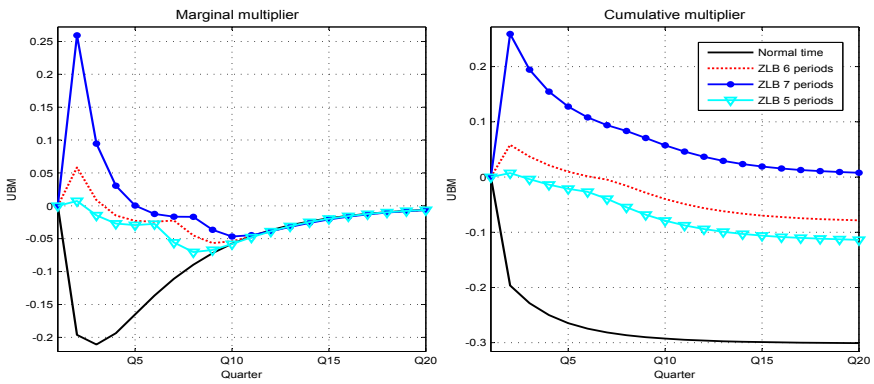

Figure 2: ***Impulse response function - Unemployment benefits multiplier.*** *Unemployment benefits increase by 1%.*

At the ZLB, a rise in unemployment benefits may increase output and lower unemployment. The intuition is as follow. Unemployment benefits increase real wages. On one side it reduces the search intensity and the incentive for firms to hire. But on the other side, it curbs the decline in the marginal cost and the deflationary spiral which dampens the increase in the real interest rate. Indeed, the liquidity trap is characterized by a zero nominal interest rate and a deflation. The real interest rate is too high compared to the value that clears the market. As a consequence, the size of the multiplier hinges on 1) the size of the recessionary shock that drives deflationary spiral and 2) the ability of unemployment benefits to create a sufficient inflationary pressure to lower the real rate. If the nominal interest rate lasts several periods at the ZLB (deep recession), a fiscal policy generating inflation is likely to reduce unemployment. If the nominal interest rate lasts only few periods at the ZLB, the decrease in real interest rate does not offset the disincentive effect of unemployment benefits extensions on search and hirings. A nominal interest rate held seven periods at the ZLB increases output and reduces unemployment. The marginal multiplier becomes negative when the nominal interest rate is moving away from the ZLB as it is the case in normal circumstances. Due to the persistence of the unemployment benefits shock the cumulative multiplier is barely positive.

[6]Since $\alpha = 1$, $y_t = n_t = 1 - u_t$. Then, the decline in output of 0.25 is equivalent to a proportional rise in unemployment.

4.3 Counterfactual analysis

We now perform a counterfactual analysis based on the observed unemployment path. We evaluate what would have been the path of the economy, and especially unemployment, in the absence of the ZLB, the unemployment benefits extension and both at the same time. First, we build a time series for the replacement ratio based on variations in the maximum benefits eligibility duration (see Figure 3). The benefits extension is based on two programs: the Extended Benefits program (EB) and the Emergency unemployment compensation (EUC08)[7]. It is worth mentioning that the maximum duration strongly increased but never exceeded 90 weeks when aggregated over each states.

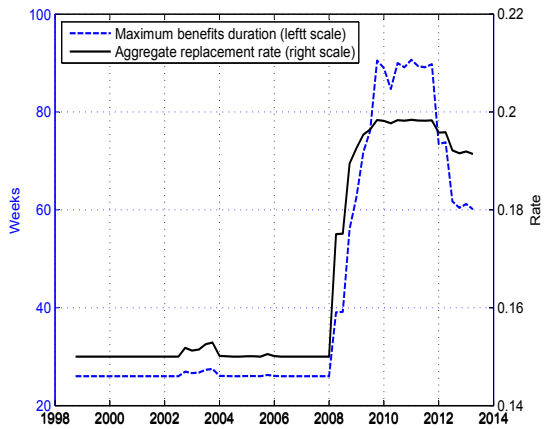

Figure 3: ***Unemployment insurance replacement rate.***

We compute the shock τ_t that reproduces the replacement rate. Second, we solve for the path of the discount factor shock that makes the simulated series of the unemployment rate as closed as possible to its empirical counterpart. The path of the discount factor shock (obtained by simulations) and the replacement rate shock (calculated using the data on unemployment insurance eligibility) are depicted in Figure 4. The replacement rate increases slightly during the 2001 recession because of the automatic adjustments triggered under the Extended Benefits Program. It increases sharply in 2008 according to the Extended Benefits Program and the Emergency Unemployment Compensation. The discount factor shock is well bellow zero at the beginning of the sample since the unemployment rate is quite low before the 2001 recession. It almost came back to zero and falls short again as the unemployment recovers. Finally, the huge increase in unemployment calls for a high discount factor and gently decreases as unemployment declines.

[7]See appendix for details.

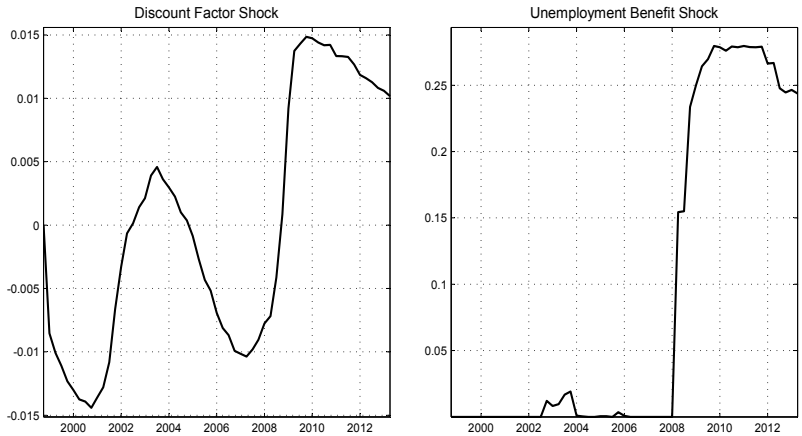

Figure 4: ***Path of the shocks.***

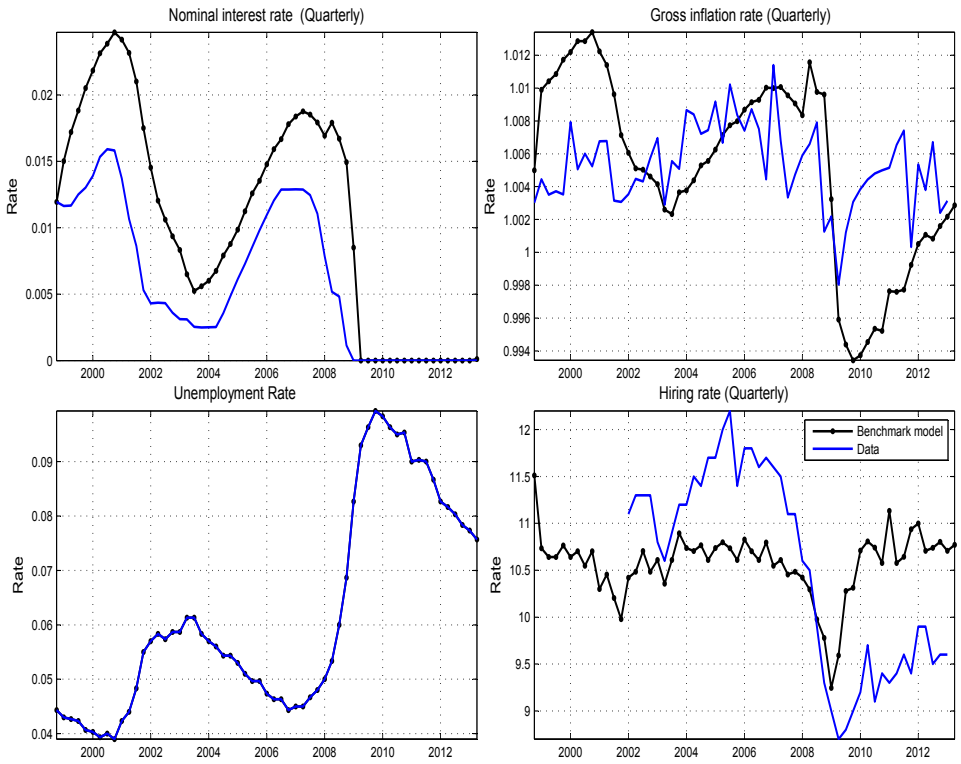

Figure 5: ***Simulated series vs Data.***

Figure 5 shows the simulated path for macroeconomic time series. Despite the remarkable simplicity of the model, the fit of the aggregate variables is surprisingly good. Recall that we only use one shock to reproduce the historical unemployment rate, the rest of the macroeconomic series being simulated. The simulated nominal interest rate behaves similarly to the one observed in

the data. It hits the ZLB just one quarter later but stays at the ZLB until now. The model matches reasonably well the magnitude of the observed deflation. The simulated fall in the hiring rate in 2008 is not as severe as in the data but still acceptable.

The alternative scenarios are presented in Figure 6. Given the path of the shocks, an economy without ZLB would have experienced lower unemployment. Differently said, a negative nominal interest rate instead of the 0% would have implied a decrease in unemployment. The biggest fall in unemployment that an unconstrained Taylor rule would have permitted is in 2011. A downward adjustment in the nominal interest rate to -1.3% would have dampened the rise in unemployment by about 1 percentage points. While the ZLB has a small impact on inflation, the simulations suggest that the hiring rate could have recovered their long-run level more rapidly.

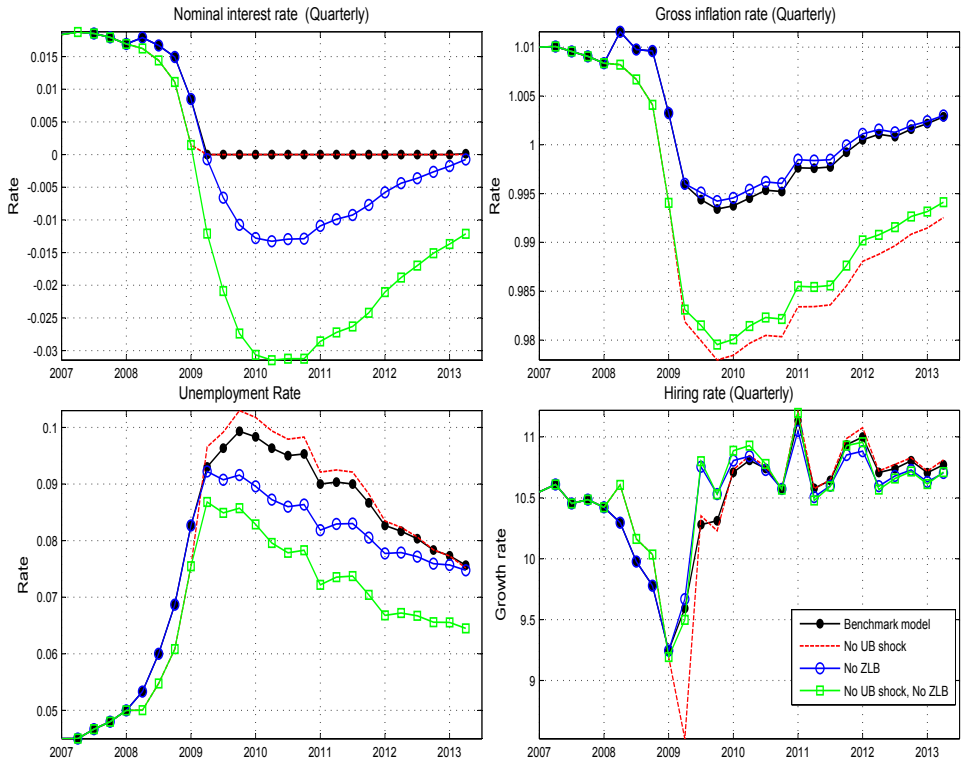

Figure 6: ***Counterfactual analysis.***

The second counterfactual scenario (red dotted line) consists in simulating the model using the path of the discount factor shock but not the replacement rate shock. In line with the intuition of previous experiments, the unemployment benefits extension is far from having exacerbate the rise in unemployment. As the path of inflation suggests, the inflationary pressure caused by the rise in the replacement rate breaks the real interest rate increase. In turn, this effect offset the decline in search intensity and the rise in real wages that

make employers less prone to post vacancies. Unemployment would have been lower in 2009Q1 since the nominal interest rate was not at the ZLB in our simulation. Thereafter, unemployment would have been higher if the US had not triggered the EB and the EUC08.

Last but not least, we remove both the ZLB and the variation in unemployment benefits in order to highlights the impact of unemployment benefits in the absence of the ZLB. The implied larger deflation, would have called for negative interest rate (-3% quarterly). If the interest rate was allowed to fall below zero, the unemployment rate would have declined by about 2 percentage points (green squared line). The reason is that making τ_t constant would have translated into a downward pressure on real wages without raising the real interest rate. Consequently, the increase in employers' surplus would have fostered the job creation otherwise. This is nothing else than the effect of unemployment benefits in normal circumstances. We conclude that extended benefits reduce unemployment but only at the ZLB.

4.4 Robustness of results

We check the robustness of the major result *i.e* the increase in unemployment when no benefits extensions are triggered at the ZLB. We use alternative specifications of the replacement rate shock, different calibrations for the search intensity curvature, the cost of posting vacancies, the level of matching frictions and we introduce real wage rigidities. Figure 7 shows the variation in unemployment (w.r.t the observe rate) when the UI extension is shut down. In most of the cases, unemployment would have declined early in the crisis. The reason is that the rise in unemployment benefits have been triggered before the nominal interest rate reaches the ZLB. The standard effects of unemployment benefits apply. Differently said, if the goal of policy makers was to lower unemployment, either the central bank did not adjusted the nominal interest rate downward rapidly enough, either the rise is unemployment benefits was too early. We now focus on the ZLB.

We define the replacement rate as the government expenditures in unemployment insurance per unemployed divided by the aggregate wage rate[8]. The increase in unemployment induced by the unresponsiveness of UI benefits would have been quite similar to the initial case. This result holds when considering the cyclical component of the replacement rate as a proxy for the shock[9]. As an additional exercise, we assume that b_t does not depend on wages: $b_t = b\exp(g_t)$, $g_t \sim AR(1)$. We set b so as to make the steady state unchanged. As previously, we remove the simulated shock that reproduces the replacement rate in level and in deviation from the trend. UI benefits still

[8]See appendix C for details.

[9]We remove the trend using an HP-filter with smoothing parameter 1600.

lower the increase in unemployment during the liquidity trap.

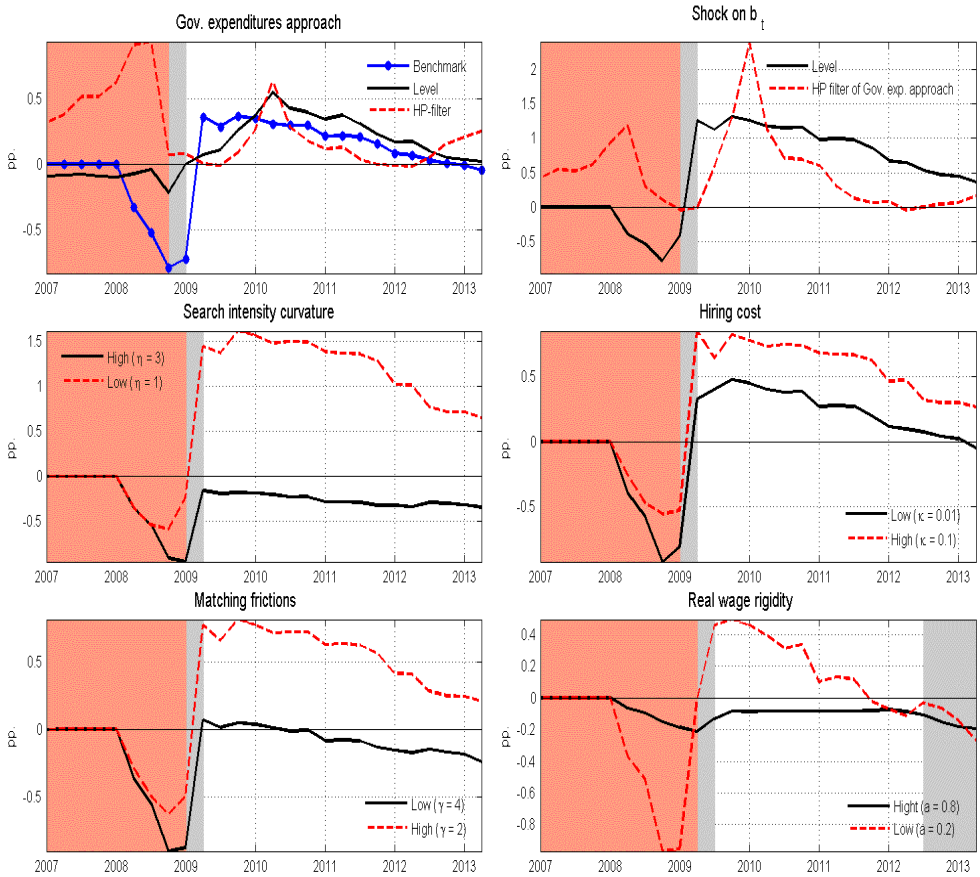

Figure 7: ***Counterfactual analysis: Robustness of results***. *Variation in unemployment in pp. when the UI extension is shut down. Red shaded area: outside the ZLB for the red dotted line. Gray shaded area: outside the ZLB for the black solid line.*

We now set the search intensity curvature η equal to 1 (as in Merz (1995)) and to 3. The search intensity may have a sizeable impact on unemployment. We observe an average 0.25 percentage point decline in unemployment if $\eta = 3$. In Merz (1995) calibration, the unemployment benefits extension would have avoided an additional 1.5 percentage points increase in unemployment in 2009. It should be noted that if search effects have little incidence, as suggested by Chetty (2008) and Hagedorn et al. (2013), the decrease in unemployment induced by the policy could be bigger.

The contribution of labor market frictions can be determined by changing κ and γ. Not surprisingly, more matching frictions (γ lower) or more hiring costs (κ higher) are likely to increase the unemployment gain from the fiscal intervention. Indeed, the mismatches between workers and jobs become higher. The policy translates more into wages than into quantities (labor flows). It re-

sults in a stronger wage pressure, which is worthwhile to lower real interest rate. Even with a low degree of labor market frictions, we do not observe any sizeable decrease in unemployment.

Finally, we introduce a real wage rigidity in the form of a wage norm as in Shimer (2005):

$$w_t^* = aw^R + (1-a)w_t^R \tag{29}$$

where a stands for the degree of real wage rigidity and w_t^* is the wage rate in the economy. It is a weighted sum of the Nash bargained wage w_t^R and a constant (the steady state level). Basically, more wage rigidities reduce the inflationary pressure but also the negative impact of wage on firms' incentive to post vacancies. The two effects canceled out. As long as the interest rate is stacked at ZLB, the policy has virtually no impact when the degree of wage rigidity is high.

5 Conclusion

The novelty of the present paper is to investigate the extent to which the unprecedent unemployment benefits extension is responsible for the strong increase in unemployment during the Great Recession. The disincentive aspect of UI has been frequently pointed out as an important source of excessive unemployment but has never been studied in an economy characterized by high real interest rate. We argue that the interaction between the unemployment benefits extension and the ZLB matters. We show that the ZLB may amplify the increase in unemployment following a demand-driven recession. Unemployment benefits extensions cause high unemployment only if the economy is out of the liquidity trap. When the nominal interest rate is held at the ZLB, its impact on firms willingness to hire is ambiguous. It depends on whether the inflationary pressure that reduces real interest rate offset the disincentive effect of unemployment benefits extensions on hirings.

For a broad variety of calibrated parameters it is shown that the UI extension, even in the worst case considered, does not cause a sizeable increase in unemployment as long as the interest rate is held at the ZLB. Using a standard calibration, a rise in unemployment benefits reduces, not increases, unemployment. These results go in opposite directions to previous studies that abstract from the liquidity trap effects. More generally, we highlight the importance of the macroeconomic conditions, especially the behavior of the interest rate and the aggregate demand, for a quantitative evaluation of unemployment benefits extensions.

We are aware that the present contribution uses a very simple and probably too stylized model. It can be extended in many directions in order to provide

a better robustness analysis of UI extensions. It seems crucial to investigate alternative wage structures, the role of labor market participation, the behavior of layoff, the heterogeneity among workers and firms and additional sources of frictions as in Carrillo & Poilly (2013). In addition, we have left completely aside the normative implications. The welfare gains (losses) could imply different policy recommendations. For instance, Mitman & Rabinovich (2011) show that the path of optimal unemployment benefits is pro-cyclical while Moyen & Stahler (2012) and Landais et al. (2010) found the opposite. Rendahl (2012) shows that a government spending policy in a severe recession characterized by a ZLB involves a huge fiscal multiplier, reduces unemployment and is unambiguously Pareto improving. However, these issues are beyond the scope of this paper.

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A Unemployment insurance benefits extension

States unemployment insurance and the federal government have adjusted unemployment benefits through the duration margin. In normal circumstances and under the regular program: *Unemployment Compensation (UC)*, an eligible unemployed worker may receive unemployment benefits up to 26 weeks in most states. During economic downturn, automatic benefits extensions are triggered under the *Extended Benefits (EB)* program. The duration is 13 or 20 weeks depending on the state's insured unemployment rate (IUR) or the total unemployment rate (TUR). In addition, the *Emergency Unemployment Compensation (EUC08)* has been launched in 2008 and has been redefined in the ARRA context in 2009. It also increases the maximum benefits duration. Four waves called « Tiers » have been implemented. The first one (Tiers I) is effective without any conditions on states' experience with unemployment. Tiers II, III and IV require a condition on the IUR and/or TUR to be effective.

For these purposes, we extract the series of the IUR and TUR for each state[10] of the US and compute if the state is eligible for the EB and the EUC08 programs. The sum of these three programs gives the maximum duration of unemployment benefits for each state. It is weighted in order to build an aggregate indicator. We assume the weights are equal to the number of total insured unemployed workers in the state divided by the total insured unemployed workers in the US. The next step is to build a time series for unemployment benefits. We assume that the extended UI benefits, characterized by a strong increase in the maximum benefits duration, can be viewed as an increase in the aggregate replacement rate. Indeed, an increase in the duration involves a higher expected value from unemployment. Since the mechanisms behind the labor market mainly rely on the expected marginal value of employment, any rise in the benefits duration acts like a decrease in the marginal

[10]We use data from each state except Virgin Island for which we do not have the entire time series. We therefore have 52 states.

value. For this reason, a shock on the replacement rate in our model seems to be a good and tractable candidate to proxy the extended unemployment insurance benefits[11]. In order to build this shock we exploit the variations in the maximum unemployment benefits duration[12] calculated previously. The use of unemployment benefits duration to determine a replacement rate index is widely used by the OECD and Layard et al. (2005) for international comparisons. Basically, they determine the benefit replacement ratio by the following formula:

$$\tau = \frac{0.6(2^{nd}\text{and } 3^{rd}\text{year RR}) + 0.4(4^{th}\text{and } 5^{th}\text{year RR})}{(1^{st}\text{year RR})}$$

where RR stands for replacement ratio. RR is equal to the gross replacement rate τ^g (50% in our case) multiplied by the number of weeks the unemployed worker receives unemployment benefits during the j-th year ($j = 1, ..., 5$) of unemployment divided by 53. This methodology determines a replacement rate through a duration, which is useful for our purpose. In addition, it gives less weight on durations exceeding 3 years. The RR in year 2 to year 5 are divided by the first year RR. This takes into account the declining profile of benefits. We adapt these two assumptions because we do not perform any international comparisons and unemployment benefits do not decrease with unemployment spells as long as the unemployed worker is eligible. We adapt our calculation by focusing on the first 8 quarters because the maximum duration never exceeds this spell and we assume quarterly frequencies. We also adjust the weights every quarters in a decreasing fashion. We denote by τ_t^i the replacement rate in state i and d_i the maximum duration of unemployment benefits in state i. The aggregate replacement rate τ_t in the US is defined as:

$$\tau_t^i = \eta\, \tau^g \frac{1}{13} \sum_{j=1}^{8} \omega_j \min(\max(d_i - (j-1)13, 0), 13) \tag{30}$$

$$\tau_t = \sum_{i=1}^{52} \frac{U^i}{U^{us}} \tau_t^i \tag{31}$$

We use min max function times $\frac{1}{13}$ because it defines the fraction of the j-th quarter in which the insured unemployed receives benefits. If $d_i > 13(j-1)$, the fraction is simply one. This ensures that $\tau_t^i \in [0, 1]$ and corresponds to the OECD methodology. η denotes the fraction of insured unemployed workers

[11] A change in the replacement rate can be viewed as a fall in the transition rate between short-term (eligible to unemployment benefits) unemployed workers and long-term unemployed (not eligible to unemployment benefits) because it affects the average outside option of a worker in the wage bargaining.

[12] It is worth noting that the gross replacement rate has remained virtually unchanged and is about 50%, depending on the familial situation.

over the total unemployment workers. We take the US average rate which is equal to 0.4 over 1998Q1-2013Q2. This account for the fraction of unemployed workers that are not eligible for unemployment benefits. Recall that we focus on the average replacement rate. $\tau^g = 0.5$ is the gross replacement ratio at the steady state. $\omega_j = 0.5^j$ with $j = 1, ..., 8$ denote the weighting functions. Obviously, alternative functions can be considered but this basic function put more weight on the first quarters, in line with the OECD methodology. Since no maximum benefits spell exceed 104 weeks, $\omega_j = 0.5^j$ is well suited because $\sum_j \omega_j$ is close to one. U^i and U^{us} denote the number of insured unemployed in state i and in the US respectively. The maximum benefits duration in the EB and EUC08 programs is calculated using the IUR and TUR state eligibility conditions reported in Table 2 and in Table 3 respectively. The duration and the replacement rate are depicted in Figure 3.

Table 2: **EXTENDED BENEFITS PROGRAM (EB)**

The 1st 13 weeks of EB can be triggered on via a 5% 13 week IUR if the IUR is equal to or greater than 120% of the prior 2 years' averaged IUR over the same 13 week period OR via a 6% 13 week IUR with no lookback. The 6% IUR option however is optional and as of now 12 states do not have this option enabled. The TUR triggers are completely optional and while most states do have the TUR option in effect now, many simply adjusted their law when EB was made 100% federally funded and have plans to drop it once 100% federal funding -2is over at the end of CY 2013. Those triggers are 6.5% and 110% of any of the prior 3 years (reverts to the prior 2 years at the end of CY 2013 as well) for 13 weeks of benefits and 8% and 110% of any of the prior 3 years (also reverts to 2 at end of CY 2013) for 20 weeks of benefits. The 5% IUR at 120% average of the prior 2 years corresponding IUR's is not optional, while all other triggers are. As mentioned above, many states adopted the TUR trigger options when congress made EB 100% federally funded back in 2009. If congress does not extend that provision most of those states have plans to drop the TUR option.

Source: DOLETA

Finally, the shock series are determined using the AR(1) (written in a different manner):

$$\begin{aligned} b_t^R &= \tau_t w_t^R \\ b_t^R &= \tau \exp(g_t) w_t^R \quad \text{Where } g_t = \rho_g g_{t-1} + \varepsilon_t^g \end{aligned}$$

The shock g_t must adjust to match the replacement rate τ_t^{data} obtained previ-

ously:

$$\begin{aligned} \tau \exp(g_t) &= \tau_t^{data} \\ g_t &= \log\left(\frac{\tau_t^{data}}{\tau}\right) \end{aligned}$$

τ is set to be the minimum value of τ_t^{data}. Then, we assume the shock on the replacement rate is initially equal to zero but each time positive. Assuming that $\tau > \min(\tau_t^{data})$ does not change our main results.

Table 3: **EMERGENCY UNEMPLOYMENT COMPENSATION 2008 (EUC08)**

Start Date	Program Extension of EUC08	End Date	Maximum Weeks
July 2008	13 weeks for all states	November 2008	13
November 2008	Tier I - 20 weeks for all stats Tier II - 13 weeks for states with TUR $>$ 6%	November 2009	20 + 13
November 2009	Tier I - 20 weeks for all states Tier II - 14 weeks for all states Tier III - 13 weeks if states TUR $\geq$ 6% Tier IV - 6 weeks if states TUR $\geq$ 8.5%	February 2012	20 + 14 + 13 + 6
February 2012	Tier I - 20 weeks for all states Tier II - 14 weeks for all states Tier III - 13 weeks if states TUR $\geq$ 6% Tier IV - 6 weeks if states TUR $\geq$ 8.5% (16 weeks if no active EB and TUR $\geq$ 8.5)	September 2012	20 + 14 + 13 + 6
May 2012	Tier I - 20 weeks for all states Tier II - 14 weeks if states TUR $\geq$ 6% Tier III - 13 weeks if states TUR $\geq$ 7% Tier IV - 6 weeks if states TUR $\geq$ 9%	September 2013	20 + 14 + 13 + 6
September 2012	Tier I - 14 weeks for all states Tier II - 14 weeks if states TUR $\geq$ 6% Tier III - 9 weeks if states TUR $\geq$ 7% Tier IV - 10 weeks if states TUR $\geq$ 9%	December 2013	14 + 14 + 9 + 10

Source: DOLETA, Whittaker & Isaacs (2013)

B Data sources

Variables	Type	Source	Code
Unemployment rate	Rate, s.a, 16 years and over	Bureau of Labor Statistics (BLS)	LNS14000000
Vacancies rate	Rate, s.a, Job openings /(employment plus job openings). Total nonfarm	Bureau of Labor Statistics (BLS)	JTS00000000JOL
Hirings rate	Rate, s.a, total nonfarm Hirings/Employment	Bureau of Labor Statistics (BLS)	JTS00000000HIR
Nominal interest rate	Rate, Effective Federal Funds Rate, Monthly, Not s.a	Federal Reserve Bank of St. Louis (FRED)	FEDFUNDS
Gross inflation rate	Rate, GDP: Implicit Price Deflator, Quarterly. s.a Index 2005=100, Growth	Federal Reserve Bank of St. Louis (FRED)	GDPDEF
Insured Unemployment	Number Weekly Claims Report Quarterly, Average, s.a	Department of Labor & Training Administration (Doleta)	CCSA
Total Unemployed	Thousands of Persons Quarterly, average, s.a	Bureau of Labor Statistics (BLS)	UNEMPLOY
Labor Force	Level, Civilian 16 years and over Number in thousands	Federal Reserve Bank of St. Louis (FRED)	LNS11000000
Wages	Level, Total Wages Quarterly, not s.a	Department of Labor Training Administration (Doleta)	TOTWAGE
Unemployment Benefits	Government social benefits to persons, UI Quarterly, Sum, s.a	U.S. Department of Commerce Bureau of Economic Analysis (BEA)	W825RC1

Table 4: ***Data source and definitions.***

C Alternative definition of the replacement rate

We take the total government expenditures in unemployment benefits (W825RC1) divided by the number of insured unemployed workers. We obtain a measure for unemployment benefits per insured unemployed worker (in$). We multiply this value by the fraction of insured unemployed workers in total unemployed workers. The amount is divided by the average wage per employed woker which is equal to the total wage divided by the number of employed workers. One gets a measure of the replacement based on government expenditures (see Figure 8).

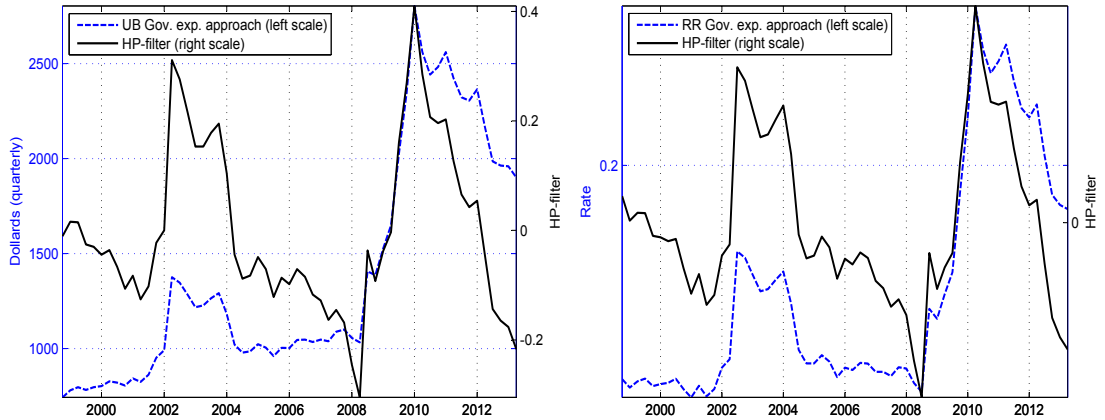

Figure 8: ***Alternative measures of the replacement rate and unemployment benefits.***

Technical Appendix (Not in the paper)

D Unemployment benefits multiplier

D.1 Marginal multiplier

Following Fernández-Villaverde et al. (2012) and Albertini et al. (2014), the marginal multiplier corresponds to the deviation of output to its long-run value divided by the initial increase in unemployment benefits. We denote by UBM_t the unemployment benefits multiplier. In normal time, it is equal to

$$UBM_t = \frac{y_{\tau,t} - y}{b_{\tau,1} - b}$$

where y, b, $y_{\tau,t}$ and $b_{\tau,1}$ are the output and the unemployment benefits long-run values, the response of output and the unemployment benefits b_t to a shock on τ_t in period 1 (upon the shock) respectively. To compute the multiplier at the ZLB we force the economy to enter in a liquidity trap for 6 periods using the discount factor shock β_t. It involves a shock on β of $\sigma_\beta = 0.02$. Let $y_{\tau,\beta,t}$ be the response of output following a shock on τ_t and β_t. $y_{\beta,t}$ and $b_{\beta,t}$ are the response of output and unemployment benefits to a shock on β_t only. The multiplier at the ZLB is equal to:

$$UBM_t^{ZLB} = \frac{y_{\tau,\beta,t} - y_{\beta,t}}{b_{\tau,\beta,1} - b_{\beta,1}}$$

The size of the shock b_t is set to 1%. Since the fiscal shock is temporary, output deviates from its unconditional mean on impact and returns to its long-run level in the absence of additional shock as time goes by. This calculation involves that the multiplier is computed on a constant basis (the increase in b_t) and ensures that the UBM returns to zero in the long-run. In order to investigate the whole gains (losses) from a fiscal intervention we analyze the cumulative multiplier which better highlights the long-run effects.

D.2 Cumulative multiplier

Following Leeper et al. (2009) and Uhlig (2010) the cumulative multiplier corresponds to the net present value of output gains relative to the increase in unemployment benefits. Output deviation from its long-run value is summed up to that date and discounted with the steady state nominal interest rate. It is divided by unemployment benefits variations discounted in a similar way. Formally it writes,

$$cUBM_t = \frac{\sum_{j=1}^{t} R^{-j}(y_{\tau,j} - y)}{\sum_{j=1}^{t} R^{-j}(b_{\tau,j} - b)}$$

The main difference here is that both terms are discounted using the steady state gross nominal interest rate $R_t = 1 + i_t$. The cumulative multiplier at the ZLB is computed in the same way except that we allow for an additional shock to force the economy to enter in a liquidity trap:

$$cUBM_t^{ZLB} = \frac{\sum_{j=1}^{t} R^{-j}(y_{\tau,\beta,j} - y_{\beta,j})}{\sum_{j=1}^{t} R^{-j}(b_{\tau,\beta,j} - b_{\beta,j})}$$

E Summary equations

E.1 SaM model

E.1.1 Dynamic model

• Euler equations:

$$\lambda_t = c_t^{-\sigma} \tag{32}$$

$$\lambda_t = (1 + i_t) E_t \beta_{t+1} \frac{\lambda_{t+1}}{\pi_{t+1}} \tag{33}$$

• Law of motion of employment:

$$n_t = (1 - \rho^x) n_{t-1} + m_t \tag{34}$$

• Hirings:

$$m_t = \left((e_t s_t)^{-\gamma} + v_t^{-\gamma} \right)^{-\frac{1}{\gamma}} \tag{35}$$

• Job seekers:

$$s_t = 1 - (1 - \rho^x) n_{t-1} \tag{36}$$

• Unemployment rate:

$$u_t = 1 - n_t \tag{37}$$

• Job finding rate:

$$f_t = \frac{m_t}{e_t s_t} \tag{38}$$

• Job filling rate:

$$q_t = \frac{m_t}{v_t} \tag{39}$$

• Worker expected marginal value of employment

$$\varphi_t = \lambda_t \left(w_t^R - b_t^R\right) - \ell n_t^\phi \tag{40}$$

$$+E_t\beta_{t+1}(1-\rho^x)\left(k(e_{t+1})\lambda_{t+1} + (1-f_{t+1}e_{t+1})\varphi_{t+1}\right) \tag{41}$$

• Firm expected marginal value of employment

$$\mu_t = mc_t\alpha\frac{y_t}{n_t} - w_t^R + (1-\rho^x)E_t\beta_{t+1}\frac{\lambda_{t+1}}{\lambda_t}\mu_{t+1} \tag{42}$$

• Free entry condition

$$q_t\mu_t = \kappa \tag{43}$$

• Output:

$$y_t = n_t^\alpha \tag{44}$$

• Marginal cost:

$$mc_t = \left(\frac{n_t}{\alpha y_t}\right)\left(w_t^R + \frac{\kappa}{q_t} - (1-\rho^x)E_t\beta_{t+1}\frac{\lambda_{t+1}}{\lambda_t}\frac{\kappa}{q_{t+1}}\right) \tag{45}$$

• Real wage:

$$w_t^R = \xi\left(b_t^R + \frac{\ell n_t^\phi}{\lambda_t} - E_t\beta_{t+1}(1-\rho^x)\frac{\lambda_{t+1}}{\lambda_t}k(e_{t+1})\right) \tag{46}$$

$$+ (1-\xi)\left(mc_t\alpha\frac{y_t}{n_t} + E_t\beta_{t+1}(1-\rho^x)\frac{\lambda_{t+1}}{\lambda_t}\kappa\theta_{t+1}e_{t+1}\right) \tag{47}$$

• Search effort:

$$\lambda_t k'(e_t) = \varphi_t f_t \tag{48}$$

• NKPC:

$$0 = (1-\epsilon) + \epsilon mc_t - \psi\frac{\pi_t}{\pi}\left(\frac{\pi_t}{\pi} - 1\right)$$

$$+ E_t\beta_{t+1}\frac{\lambda_{t+1}}{\lambda_t}\psi\frac{\pi_{t+1}}{\pi}\left(\frac{\pi_{t+1}}{\pi} - 1\right)\frac{y_{t+1}}{y_t} \tag{49}$$

• Taylor rule:

$$1+i_t = \begin{cases} (1+i_t)^{\rho_i}\left(\frac{\pi}{\beta}\left(\frac{\pi_t}{\pi}\right)^{\rho_\pi}\left(\frac{y_t}{y}\right)^{\rho_y}\right)^{1-\rho_i} & \text{if} \quad i_t > 0 \\ 1 & \text{otherwise} \end{cases} \tag{50}$$

• Unemployment benefits:

$$b_t^R = \tau_t w_t^R \tag{51}$$

• Market clearing:

$$y_t\left[1-\frac{\psi}{2}\left(\frac{\pi_t}{\pi}-1\right)^2\right] = c_t + \kappa v_t + k(e)s \tag{52}$$

• Shocks:

$$\log\beta_{t+1} = \rho_\beta \log\beta_t + (1-\rho_\beta)\log\beta + \sigma_\beta\varepsilon_{t+1}^\beta \tag{53}$$
$$\log\tau_{t+1} = \rho_\tau \log\tau_t + (1-\rho_\tau)\log\tau + \sigma_\tau\varepsilon_{t+1}^\tau \tag{54}$$
$$\tag{55}$$

E.1.2 Steady state model

• Euler equations:

$$\lambda = c^{-\sigma} \tag{56}$$
$$(1+i) = \frac{\pi}{\beta} \tag{57}$$

• Law of motion of employment:

$$n = \frac{fs}{1-(1-\rho^x)} \tag{58}$$

• Hirings:

$$m = \left((es)^{-\gamma} + v^{-\gamma}\right)^{-\frac{1}{\gamma}} \tag{59}$$

• Job seekers:

$$s = 1-(1-\rho^x)n \tag{60}$$

• Unemployment rate:

$$u = 1 - n \tag{61}$$

• Job finding rate:

$$f = \frac{m}{es} \tag{62}$$

• Job filling rate:

$$q = \frac{m}{v} \tag{63}$$

• Worker expected marginal value of employment

$$\varphi = \lambda \left(w^R - b^R \right) - \ell n^\phi + \beta(1-\rho^x)(k(e)\lambda + (1-fe)\varphi \tag{64}$$

• Firm expected marginal value of employment

$$\mu = mc\alpha \frac{y}{n} - w^R + (1-\rho^x)\beta\mu \tag{65}$$

• Free entry condition

$$q\mu = \kappa \tag{66}$$

• Output:

$$y = n^\alpha \tag{67}$$

• Marginal cost:

$$mc = \left(\frac{n}{\alpha y} \right) \left(w^R + \frac{\kappa}{q} - (1-\rho^x)\beta \frac{\kappa}{q} \right) \tag{68}$$

• Real wage:

$$w^R = (1-\xi) \left(mc\alpha \frac{y}{n} + (1-\rho^x)\beta\kappa\theta e \right) + \xi \left(b^R + \frac{\ell n^\phi}{\lambda} - \beta(1-\rho^x)k(e) \right) \tag{69}$$

• Search effort:

$$k'(e) = \frac{\varphi}{\lambda} f \tag{70}$$

• NKPC:

$$mc = \frac{\epsilon - 1}{\epsilon} \tag{71}$$

• Taylor rule:

$$1 + i = \frac{\pi}{\beta} \tag{72}$$

• Unemployment benefits:

$$b^R = \tau w^R \tag{73}$$

• Market clearing:

$$y = c + \kappa v + k(e)s \tag{74}$$

E.1.3 Calibration

With $\pi = 1.0055$ and $\beta = 0.994$ then,

$$i = \frac{\pi}{\beta} - 1 = 1.15\%$$

In the benchmark we impose the following values: $\alpha = 1$, $\sigma = 1$, $\phi = 1$ and $\tau = 0.15$. We set the steady state level of employment to 0.945, $\rho^x = 0.1061$ and $e = 1$.

$$\begin{aligned} m &= \rho^x n = 0.10 \\ s &= 1 - (1 - \rho^x)n = 0.16 \\ f &= \frac{m}{se} = 0.65 \\ y &= n^\alpha = 0.945 \end{aligned}$$

Since $q = 0.9$ then

$$\begin{aligned} v &= \frac{m}{v} = 0.11 \\ \theta &= \frac{f}{q} = 0.74 \end{aligned}$$

γ is set to balance the hiring function given m=0.10. Then $\gamma = 2.741$. We can find c since we set $\kappa = 0.05$

$$c = y - \kappa v - k(e)s = 0.93929$$

Using it in the Euler equation we get:

$$\lambda = (y - \kappa v - k(e)s)^{-\sigma} = 1.06$$

Using $\epsilon = 6$ and $mc = (\epsilon - 1)/\epsilon \simeq 0.83$ and the job creation condition we find w^R

$$w^R = mc\alpha\frac{y}{n} + \frac{\kappa}{q} - (1-\rho^x)\beta\frac{\kappa}{q} = 0.83$$

given $\tau = 0.15$ and $b^R = \tau * w^R = 0.12$, one has to determine ℓ that satisfy the real wage equation:

$$\ell = \frac{\lambda}{\xi n^{\phi}}\left(w^R - (1-xi)\left(mc\alpha\frac{y}{n} + \beta(1-\rho^x)\kappa\theta\right) - \xi b\right) = 0.77$$

Given that $k(e) = k_0 e^{\eta}$ and η is set to 2, k_0 is obtained using the free entry condition and the Nash bargaining rule:

$$k_0 = \frac{1-\xi}{\eta\xi e^{\eta-1}}\kappa\theta = 0.018$$

F Solution method

In order to explicitly take into account the non-linearity induced by the ZLB we use a projection method: Parameterized Expectation Algorithm (PEA). It consists in approximating the policy rules and conditional expectations of the system previously described using Chebyshev polynomials. These parametric functions display suitable orthogonality and convergence properties to minimize the error distance approximation. We consider a third-order Chebyshev polynomial over a fixe complete grid. Our strategies is all the more accurate that we approximate two policy functions: the ZLB and outside the ZLB. We merge them in the algorithm to compute the expectations. Then, one can approximate kink in all decision rules accurately. We first present some numerical technics that will be helpful for the understanding of the general algorithm[13].

F.1 Model dynamic equation

Let first summarize the model equations. The competitive equilibrium in a reduced form bloc can be defined as follow:

[13]This appendix does not intend to be a mathematical note on the algorithm but a general description on the way the model is solved using Matlab. For this reason, many variables will be treated as vectors or matrices, as we do in Matlab.

Backward looking dynamics

$$
\begin{aligned}
1+i_t &= (1+i_{t-1})^{\rho_i}\left(\frac{\pi}{\beta}\left(\frac{\pi_t}{\bar{\pi}}\right)^{\rho_\pi}\left(\frac{y_t}{y}\right)^{\rho_y}\right)^{1-\rho_i} \\
n_t &= (1-\rho^x)n_{t-1}+v_t q_t \\
\log\beta_{t+1} &= \rho_\beta\log\beta_t+(1-\rho_\beta)\log\beta+\sigma_\beta\varepsilon^\beta_{t+1} \\
\log\tau_{t+1} &= \rho_\tau\log\tau_t+(1-\rho_\tau)\log\tau+\sigma_\tau\varepsilon^\tau_{t+1}
\end{aligned}
$$

Forward looking dynamics

$$
\begin{aligned}
\lambda_t = (1+i_t)E_t\Upsilon^1_{t+1} \quad &\text{if} \quad i_t>0 \\
\lambda_t = E_t\Upsilon^1_{t+1} \quad &\text{if} \quad i_t=0
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\kappa}{q_t} &= mc_t\alpha\frac{y_t}{n_t}-w^R_t+(1-\rho^x)E_t\frac{\Upsilon^2_{t+1}}{\lambda_t} \\
w^R_t &= (1-\xi)\left(mc_t\alpha\frac{y_t}{n_t}+E_t\frac{\Upsilon^3_{t+1}}{\lambda_t}\right)+\xi\left(b^R_t+\frac{\ell n^\phi_t}{\lambda_t}\right) \\
0 &= (1-\epsilon)+\epsilon mc_t-\psi\frac{\pi_t}{\pi}\left(\frac{\pi_t}{\pi}-1\right)+E_t\frac{\Upsilon^4_{t+1}}{y_t\lambda_t}
\end{aligned}
$$

$$
k_0\eta e_t^{\eta-1}=\frac{1-\xi}{\xi}\kappa\theta_t
$$

where the intermediary variables are (in the compact form model):

$$
\begin{aligned}
m_t &= \left((e_ts_t)^{-\gamma}+v_t^{-\gamma}\right)^{-\frac{1}{\gamma}} \\
s_t &= 1-(1-\rho^x)n_{t-1} \\
f_t &= \frac{m_t}{(e_ts_t)} \\
q_t &= \frac{m_t}{v_t} \\
y_t &= n_t^\alpha \\
c_t &= y_t(1-\Gamma^\pi_t)-\kappa v_t-s_tk_0e_t^\eta \\
\lambda_t &= c_t^{-\sigma} \\
\Gamma^\pi_t &= \frac{\psi}{2}\left(\frac{\pi_t}{\pi}-1\right)^2
\end{aligned}
$$

We define by $\Delta_t = \{i_{t-1}, n_{t-1}, \beta_t, \tau_t\}$ the vector of state variables and by : $\mathcal{E} = \{v_t, \pi_t, w_t^R, mc_t, e_t\}$ the vector control variables.

Expectation functions

$$\begin{aligned}
E_t \Upsilon_{t+1}^1 &= \beta_{t+1} \frac{\lambda_{t+1}}{\pi_{t+1}} \\
E_t \Upsilon_{t+1}^2 &= (1-\rho^x)\beta_{t+1}\lambda_{t+1}\frac{\kappa}{q_{t+1}} \\
E_t \Upsilon_{t+1}^3 &= (1-\rho^x)\beta_{t+1}\lambda_{t+1}\kappa\theta_{t+1}e_{t+1} \\
E_t \Upsilon_{t+1}^4 &= \beta_{t+1}\lambda_{t+1}\psi\frac{\pi_{t+1}}{\pi}y_{t+1}\left(\frac{\pi_{t+1}}{\pi}-1\right) \\
E_t \Upsilon_{t+1}^5 &= (1-\rho^x)\beta_{t+1}\lambda_{t+1}k(e_{t+1})
\end{aligned}$$

According to those equations, there are five policy rules to determine $\{v_t, \pi_t, w_t^R, mc_t, e_t\}$ and five expectation functions $E_t \Upsilon_{t+1}^e, e = 1, .., 5$. The basic idea of the PEA algorithm is to approximate the policy rule and the expectation function by a parametric approximation function $\Phi^p(\Delta_t)$ and $\Psi^e(\Delta_t)$ respectively, which are functions of the state vectors.

F.2 Preliminary results on Chebyshev functions

To approximate the unknown functions: the expectations and the policy rules, we rely on Chebyshev polynomials. The domain of Chebyshev polynomials is the interval [-1, 1]. We will see later on how to manage an $[a, b]$ interval. Let D be the approximation order of the Chebyshev polynomial. With one state variable x, the Chebyshev polynomials of order d is built according to the following recursion:

$$T_{d+1}(x) = 2x\, T_d(x) - T_{d-1}(x)$$

Given that $T_0(x) = 1$ and $T_1(x) = x$. Applying the trigonometric identities $T_d(\cos(x)) = \cos(nx)$ where $\cos(nx)$ is an orthogonal sequence on $[0, 2\pi]$, the d-th member of the polynomial is

$$T_d(x) = \cos(d \arccos(x))$$

The complete Chebyshev polynomial of a variable x writes:

$$\Phi(\varphi(x), \theta) = \sum_{d=0}^{D} \phi_d T_d(x)$$

where

$$\varphi(x) = 2\frac{x-a}{b-a} - 1$$

a and b are the minimum and the maximum bounds of the ergodic distribution of the variable x. Then, $\varphi(x)$ maps x in the interval [-1,1]. ϕ_d are the parameters that we have to determine to find the policy rules. When the number of state variables is higher than one, we have to build a multidimensional Chebysev polynomial. The product of polynomial terms must have an order not higher than D. Let K be the number of state variables such that $x = \{x_1, x_2, ..., x_K\}$ The multidimensional Chebyshev polynomial writes:

$$\Phi(\varphi(x), \phi) = \sum_{d_1=0}^{D} ... \sum_{d_K=0}^{D} \phi_{d_1...d_k} \mathbb{1}_{\{\sum_{s=1}^{K} d_s \leq D\}} \prod_{k=1}^{K} T_{d_k}(\varphi(x_k))$$

Where $\mathbb{1}$ is a variable taking the value 1 if the product of the $\{d_1, ..., d_D\}$-th member of the polynomial has a total order $\sum_{s=1}^{K} d_s$ not higher than D and 0 otherwise. For instance, in our model we assume a third order Chebyshev polynomial with four state variables. For each control variable j the policy rule writes:[14]:

$$\begin{aligned}
\Phi_j(\varphi(\Delta_t), \theta) &= \phi_1^j + \phi_2^j \, \varphi(i_{t-1}) + \phi_3^j \cos(2 \arccos \varphi(i_{t-1}))) \\
&+ \phi_4^j \cos(3 \arccos \varphi(i_{t-1}))) + \phi_5^j \, \varphi(n_{t-1}) \\
&+ \phi_6^j \, \varphi(n_{t-1}) \cos(2 \arccos \varphi(i_{t-1}))) + ...
\end{aligned}$$

Therefore, the Chebyshev polynomial has $\sum_{d_1=0}^{D} ... \sum_{d_K=0}^{D} \mathbb{1}_{\{\sum_{s=1}^{K} d_s \leq D\}}$ elements. It should be noted that the Chebyshev polynomial obeys the continuous orthogonality relationship:

$$\int_{-1}^{1} T_i(x) T_j(x) \sqrt{(1 - x^2)} dx = 0, \quad \text{for all} \quad i \neq j$$

This property states that the Chebyshev polynomials are orthogonal on [-1,1] with respect to the inner product defined by the weighting function: $\sqrt{(1 - x^2)}$.

F.3 Grid of the state variables

When the number of state variables is higher than one, we need to construct a multidimensional grid. We assume a complete base or "*full basis*". The complete basis is chosen instead of the popular Smolyak collocation method or tensor basis. The Smolyak collocation method (see Judd (1998), Malin et al. (2011) improves the speed of standard projection methods but it makes the convergence more difficult in a model characterized by occasionally binding constraints. Adaptive domain, cluster grid and anisotropic grid have found

[14]For the sake of clarity we normalized the subscript of coefficient θ.

a particular attention in the numerical literature (see Judd et al. (2012) and Judd et al. (2013)). However, according to the size of our model we do not need to save on the *curse of dimensionality* in the sense that the algorithm converges in less than one minutes. The complete basis provides a richer basis than the others methods which purpose is useful for large scale DSGE model. Under the tensor product basis, the nodes of the grid are at least equal to the order of the Chebyshev polynomial D plus one. We simply assume that the number of nodes is $\mathcal{N} = D + 1$. Following Barthelmann et al. (2000), we use the extrema of Chebychev polynomials as the basis for the grid points. $\mathcal{G}^{\mathcal{N}} = \{\zeta^1, ..., \zeta^{\mathcal{N}}\} \subset [-1, 1]$ is the set of the extrema of the Chebychev polynomials nodes. The roots of the Chebyshev polynomial of order D when the approximation order is the same in every dimension are:

$$\zeta^n = -\cos\left(\frac{2n-1)\pi}{2\mathcal{N}}\right) \text{ for } \quad n = 1, ..., \mathcal{N}$$

For instance, a third-order Chebyshev polynomial gives

$$\mathcal{G}^4 \;=\; \texttt{-0.9239} \quad \texttt{-0.3827} \quad \texttt{0.3827} \quad \texttt{0.9239}$$

The grid for each variable x_k at each node n is defined by:

$$\mathcal{X}_k^n = \underline{x}_k + (\zeta^n + 1)\frac{\overline{x}_k - \underline{x}_k}{2} \text{ for } \quad n = 1, ..., \mathcal{N}; \;\; k = 1, ..., K$$

where $\underline{x}_k$ and $\overline{x}_k$ denote the lower and the upper bound of the domain of the state variable x_k respectively. We now turn to the computation of the grid matrix. Let $\mathcal{I}_s$ be a vector of dimension s whose components are each equal to one. The full tensor product basis $\mathcal{H}_t$ is a set of tensor product basis for each variable k: $\mathcal{H}_t = \{\mathcal{H}^1, ..., \mathcal{H}^K\} \subset [-1, 1]$ where every $\mathcal{H}^k$ can be obtained using the Kronecker product:

$$\mathcal{H}^k = \mathcal{I}_{\mathcal{N}^{k-1}} \otimes \varphi(\mathcal{X}_k) \otimes \mathcal{I}_{\mathcal{N}^{K-k}} \qquad \text{for } k = 1, ..., K$$

$\mathcal{I}_{\mathcal{N}^{k-1}}$ and $\mathcal{I}_{\mathcal{N}^{K-k}}$ are vectors of dimension $\mathcal{N}^{k-1}$x 1 and $\mathcal{N}^{K-k}$x 1 respectively and whose elements are each equal to one. Here $\mathcal{X}_k$ is the column vector of the variable k that has $\mathcal{N}$ nodes. As previously mentioned, the function $\varphi(\mathcal{X}_k)$ maps each component of the vector $\mathcal{X}^k$ in the [-1,1] domain. $\mathcal{H}_t$ maps each combination of nodes. We add the time subscript for the grid because we will compute the next period state vector that will serve as a grid to calculate next period policy rule.

F.4 Policy rule approximations

In our model we have four control variables summarized by the vector: $\{v_t, \pi_t, w_t^R, mc_t, e_t\}$. The solution of the system maps the control variables as a function of the states

variables Δ_t. In addition, due to the kink in the Taylor rule we approximate two types of policy rules[15]: the unconstrained case characterized by $i_t > 0$ and the constrained case where $i_t \leq 0$. We define by $\Phi_g(\mathcal{H}_t)$, $g = un, c$ the policy rule. un stands for unconstrained and c for constrained.

$$\Phi_g(\mathcal{H}_t) = \left[\begin{array}{cccc} \Phi_g^1(\mathcal{H}_t) & \Phi_g^2(\mathcal{H}_t) & \Phi_g^3(\mathcal{H}_t) & \Phi_g^4(\mathcal{H}_t)\Phi_g^5(\mathcal{H}_t) \end{array} \right] \; g = un, c$$

Note that Δ_t has been replaced by $\mathcal{H}_t$ because we use the tensor grid to evaluate the policy rules. The "aggregate" policy rule combines both:

$$\Phi(\mathcal{H}_t) = \mathbb{1}_{\{i_t > 0\}} \Phi_{un}(\mathcal{H}_t) + \mathbb{1}_{\{i_t \leq 0\}} \Phi_c(\mathcal{H}_t)$$

$\mathbb{1}_{\{i_t > 0\}}$ is a vector which valuee are equal to 1 if the nominal interest rate is strictly positive and 0 otherwise. We will see later on how to calculate i_t. Let first rewrite some algebra to get a more compact form of the solution system. With $\mathcal{H}_t$ being the tensor grid, the terms in the Chebyshev function over this grid (The complete basis) writes :

$$\begin{aligned} F(\mathcal{H}_t) &= \mathbb{1}_{\{\sum_{s=1}^{K} d_s \leq D\}} \prod_{k=1}^{K} T_{d_k}(\mathcal{H}_t^k) \\ \text{where} \quad d_k &= \mathcal{I}_{D^{k-1}} \otimes \mathcal{Q} \otimes \mathcal{I}_{D^{K-k}} \qquad \text{for } k = 1, ..., K \\ \text{and} \quad \mathcal{Q} &= [1, ..., D]^T \end{aligned}$$

As before, $\mathcal{I}_{D^{k-1}}$ and $\mathcal{I}_{D^{K-k}}$ are vectors of dimension D^{k-1}x 1 and D^{K-k}x 1 respectively and which elements are each equal to one. $\mathcal{Q}$ is a vector of dimension Dx 1 (T stands for the transpose) and $F(\mathcal{H}_t)$ maps each combinations of state variables in the Chebyshev polynomial. Using this, one can rewrite the policy rules in a more compact form:

$$\Phi_g(\mathcal{H}_t, \Lambda_P^g) = F(\mathcal{H}_t)\Lambda_P^g$$

where Λ_P^g stands for the matrix coefficient of the policy rules defined by

$$\Lambda_P^g = \left[\begin{array}{cccc} \phi_{11}^g & \phi_{12}^g & \cdots & \phi_{1K}^g \\ \phi_{21}^g & \phi_{22}^g & \cdots & \phi_{2K}^g \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{S1}^g & \phi_{S2}^g & \cdots & \phi_{SK}^g \end{array} \right] \; g = un, c$$

[15]This method approximate the solution more accurately than if we use a single Chebyshev polynomial. It allows to manage the kink in the policy rules. It is important to note that the two policy rules are linked when the expectations are computed. It means that agents take into account the probability of moving in and out of the liquidity trap when they take their decisions. The probability that the nominal interest rate hits the ZLB is endogenous and taking into agents' expectations.

and S is the number of terms when the order approximation is D and the number of state variables is K. Formally it is obtained using the following formula:

$$S = \sum_{d_1=0}^{D} ... \sum_{d_K=0}^{D} \mathbb{1}_{\{\sum_{s=1}^{K} d_s \leq D\}} \qquad \text{for } k = 1, ..., K$$

F.5 Numerical integration

An important problem is the presence of two aggregate shocks (namely β_t, τ_t). The expectation functions must be evaluated. For this purpose we proceed to a numerical integration using Gauss-Hermite quadratures.

F.5.1 Gauss-Hermite quadrature

In our case, Gauss-Hermite quadrature will be particularly useful because we are dealing with stochastic processes that have gaussian distributions. Formally we have:

$$\begin{aligned} E_t G(z) &= \int_{-\infty}^{+\infty} G(x\sigma^{\varepsilon}\sqrt{2} + \mu^{\varepsilon}) e^{-x^2} dx \simeq \sum_{i=1}^{n_h} \omega_i G(x_i \sigma^{\varepsilon}\sqrt{2} + \mu^{\varepsilon}) \\ dz &= \sigma^{\varepsilon}\sqrt{2} dx \end{aligned}$$

where μ^{ε} and σ^{ε} are the mean and the variance of the shock respectively, n_h is the number of Hermite nodes (10 in our case) and ω_i are the weighting functions that are solved numerically according to:

$$\omega_i = \frac{2^{n_h+1} n_h! \sqrt{\pi}}{[H'_{n_h}(r_i)]^2}$$

where $H(.)$ is an orthogonal Hermite polynomial and r_i are the associated roots. In the presence of multiple (n_s) random variables we must compute multiple integrals:

$$\begin{aligned} E_t G(z_1, ..., z_{n_s}) &= \int_{-\infty}^{+\infty} ... \int_{-\infty}^{+\infty} G(z_1, ..., z_{n_s}) dz_1 ... dz_{n_s} \\ &\simeq \left(\frac{1}{\sqrt{\pi}}\right)^{n_s} \sum_{i_1=1}^{(n_h)^{n_s}} ... \sum_{i_{n_s}=1}^{(n_h)^{n_s}} \omega_{i_1}, ..., \omega_{i_{n_s}} G(z_{i_1}, ..., z_{i_{n_s}}) \end{aligned}$$

F.5.2 Expectations in the model

As mentioned above, the goal of the PEA is to replace expectations $E_t Y^e_{t+1}$ by a parametric approximation functions $\Psi(\Delta_{t+1})$ such that:

$$\left[\begin{array}{ccc} E_t Y^1_{t+1} & \dots & E_t Y^5_{t+1} \end{array} \right] \simeq \Psi(\mathcal{H}_t) = \left[\begin{array}{ccc} \Psi^1(\mathcal{H}_t) & \dots & \Psi^5(\mathcal{H}_t) \end{array} \right]$$

These functions are assumed to have the following representation:

$$\Psi(\mathcal{H}_t) = F(\mathcal{H}_t)\Lambda_E$$

where Λ_E stands for the matrix coefficient of the expectations:

$$\Lambda_E = \begin{bmatrix} \psi_{11} & \psi_{12} & \cdots & \psi_{1n_e} \\ \psi_{21} & \psi_{22} & \cdots & \psi_{2n_e} \\ \vdots & \vdots & \ddots & \vdots \\ \psi_{S1} & \psi_{S2} & \cdots & \psi_{Sn_e} \end{bmatrix}$$

S is the number of terms in the Chebyshev polynomial as previously described and n_e is the number of expectations (5 in our model). Note again that Δ_t has been replaced by $\mathcal{H}_t$ since we use the tensor grid to estimate Λ_E. In order to compute the expectations in the PEA procedure, one has to use the policy rules in $t+1$. To do this, we need to calculate the next period state variables, or equivalently $\mathcal{H}_{t+1}$. For endogenous state variable we use their law of motion. Given the policy rules $\Phi(\mathcal{H}_t)$ and the initial grid of states variable $\{\mathcal{H}_t^1, \mathcal{H}_t^2\}$ (being the grid of the nominal interest rate and employment respectively) the next period endogenous state vectors are:

$$\begin{aligned}
s_t &= 1 - (1-\rho^x)\mathcal{H}_t^2 \\
m_t &= ((\Phi_5(\mathcal{H}_t)s_t)^{-\gamma} + \Phi^1(\mathcal{H}_t)^{-\gamma})^{-\frac{1}{\gamma}} \\
q_t &= \frac{m_t}{\Phi_1(\mathcal{H}_t)} \\
n_t &= (1-\rho^x)\mathcal{H}_t^2 + q_t\Phi_1(\mathcal{H}_t) \\
i_t &= (1+\mathcal{H}_t^1)^{\rho_i}\left(\frac{\pi}{\beta}\left(\frac{\Phi_2(\mathcal{H}_t)}{\pi}\right)^{\rho_\pi}\left(\frac{n_t^\alpha}{y}\right)^{\rho_y}\right)^{1-\rho_i} - 1
\end{aligned}$$

where $\Phi^1(\mathcal{H}_t)$ and $\Phi^2(\mathcal{H}_t)$ denote the policy rule of vacancies and inflation respectively. The next periods exogenous state variables $\{\beta_{t+1}, \tau_{t+1}\}$ are given by:

$$\begin{aligned}
\log\beta_{t+1} &= \left(\mathcal{I}_{(n_h)^{n_s}} \otimes \rho_\beta \log\mathcal{H}_t^3 + (1-\rho_\beta)\log\beta\right) + \left(\sigma_\beta\sqrt{2}\mathcal{Z}_1 \otimes \mathcal{I}_{\mathcal{N}^K}\right) \\
\log\tau_{t+1} &= \left(\mathcal{I}_{(n_h)^{n_s}} \otimes \rho_\tau \log\mathcal{H}_t^5 + (1-\rho_\tau)\log\tau\right) + \left(\sigma_\tau\sqrt{2}\,\mathcal{Z}_2 \otimes \mathcal{I}_{\mathcal{N}^K}\right)
\end{aligned}$$

and

$$\mathcal{Z}_j = \mathcal{I}_{(n_h)^{j-1}} \otimes z \otimes \mathcal{I}_{(n_h)^{n_s-j}} \qquad \text{for } j = 1, ..., n_s$$

$\mathcal{I}_{\mathcal{N}^K}$ and $\mathcal{I}_{(n_h)^{n_s}}$ are vectors of dimension $\mathcal{N}^K$x 1 and $(n_h)^{n_s}$x 1 which components are each equal to one. n_h is the number of Hermite nodes and n_s is simply the number of exogenous stochastic processes (2 in our case). The Kronecker product of the first terms on the right-hand side $\mathcal{I}_{(n_h)^{n_s}}$ is needed to get a vector of all possible combinations in $t+1$, given that we have n_h Hermite nodes. For instance, if $\mathcal{H}_t$ is a 256x 4 matrix with four state variables, $\mathcal{H}_{t+1}$ must be a 25600x 4 matrix if, among the four state variables, there are two shocks and ten Hermite nodes. The complete basis $F(\mathcal{H}_{t+1})$ with $D = 3$ and $K = 4$ must be a 25600x 35 matrix. Given z being the n_hx1 vector of the Hermites nodes in one dimension, the multidimensional (quadratures) representation of Hermite nodes with n_s shocks ($n_s = 2$) is given by $\{\mathcal{Z}_1, \mathcal{Z}_2\}$. It is a $(n_h)^{n_s}$xn_s matrix. The next period tensor product base $\mathcal{H}_{t+1}$ is equal to

$$\begin{aligned}
\mathcal{H}_{t+1} &= \left[\mathcal{H}^1_{t+1},\ \mathcal{H}^2_{t+1},\ \mathcal{H}^3_{t+1},\ \mathcal{H}^4_{t+1}\right] \\
&= \left[\mathcal{I}_{(n_h)^{n_s}} \otimes \varphi(i_t), \mathcal{I}_{(n_h)^{n_s}} \otimes \varphi(n_t), \varphi(\log \beta_{t+1}), \varphi(\log \tau_{t+1})\right]
\end{aligned}$$

Given the next period tensor product base, we compute the next period policy rules according to $\Phi(\mathcal{H}_{t+1})$. It should be stressed that we have to compute the two types of policy rules and merge them according to the value of the nominal interest rate:

$$\Phi(\mathcal{H}_{t+1}) = \mathbb{1}_{\{i_t>0\}}\Phi_{un}(\mathcal{H}_{t+1}) + \mathbb{1}_{\{i_t\leq 0\}}\Phi_c(\mathcal{H}_{t+1})$$

Using the $t+1$ policy rules it is easy to determine the values for expectation functions Y^e_{t+1}. Each variables and expectation functions in $t+1$ become vectors of dimensions $\mathcal{N}^K(n_h)^{n_s}$x 1. For convenience, we reshape the vectors in matrices of dimensions $\mathcal{N}^K$x $(n_h)^{n_s}$. For that purpose we add a second sub-

script j to denote the $j-th$ element of the vector. Formally it writes:

$$
\begin{aligned}
\mathcal{M}_a^1 &= \prod_{j=\mathcal{N}^K(a-1)+1}^{a\mathcal{N}^K} \mathcal{H}_{t+1,j}^3 \, \frac{\lambda_{t+1,j}}{\Phi^2(\mathcal{H}_{t+1,j})} \\
\mathcal{M}_a^2 &= \prod_{j=\mathcal{N}^K(a-1)+1}^{a\mathcal{N}^K} (1-\rho^x)\mathcal{H}_{t+1,j}^3 \, \lambda_{t+1,j} \, \frac{\kappa}{q_{t+1,j}} \\
\mathcal{M}_a^3 &= \prod_{j=\mathcal{N}^K(a-1)+1}^{a\mathcal{N}^K} (1-\rho^x)\mathcal{H}_{t+1,j}^3 \, \lambda_{t+1,j} \, \kappa\theta_{t+1,j} \\
\mathcal{M}_a^4 &= \prod_{j=\mathcal{N}^K(a-1)+1}^{a\mathcal{N}^K} \mathcal{H}_{t+1,j}^3 \, \lambda_{t+1,j} \, \psi\frac{\pi_{t+1,j}}{\pi} \, y_{t+1,j} \left(\frac{\Phi^2(\mathcal{H}_{t+1,j})}{\bar{\pi}} - 1 \right) \\
\mathcal{M}_a^5 &= \prod_{j=\mathcal{N}^K(a-1)+1}^{a\mathcal{N}^K} (1-\rho^x)\mathcal{H}_{t+1,j}^3 \, \lambda_{t+1,j} \, k(e_{t+1,j}) \\
\mathcal{M}^e &= \{\mathcal{M}_1^e, ..., \mathcal{M}_{\mathcal{N}^K}^e\}
\end{aligned}
$$

where:

$$
\begin{aligned}
s_{t+1,j} &= 1-(1-\rho^x)\mathcal{H}_{t+1,j}^2 \\
m_{t+1,j} &= ((\Phi^5(\mathcal{H}_{t+1,j})s_{t+1,j})^{-\gamma} + \Phi^1(\mathcal{H}_{t+1,j})^{-\gamma})^{-\frac{1}{\gamma}} \\
q_{t+1,j} &= \frac{m_{t+1,j}}{\Phi_1(\mathcal{H}_{t+1,j})} \\
\theta_{t+1,j} &= \frac{\Phi_1(\mathcal{H}_{t+1,j})}{\Phi^5(\mathcal{H}_{t+1,j})s_{t+1,j}} \\
y_{t+1,j} &= \left(\mathcal{H}_{t+1,j}^2\right)^{\alpha} \\
\lambda_{t+1,j} &= c_{t+1,j}^{-\sigma} \\
&= \left(y_{t+1,j}\left(1-\Gamma_{t+1,j}^{\pi}\right) - \kappa\Phi^1(\mathcal{H}_{t+1,j}) - s_{t+1,j}k_0\Phi^5(\mathcal{H}_{t+1,j})^{\eta}\right)^{-\sigma} \\
&= \left(\left(\mathcal{H}_{t+1,j}^2\right)^{\alpha}\left(1-\frac{\psi}{2}\left(\frac{\Phi^2(\mathcal{H}_{t+1,j})}{\bar{\pi}}-1\right)\right) - \kappa\Phi^1(\mathcal{H}_{t+1,j}) - s_{t+1,j}k_0\Phi^5(\mathcal{H}_{t+1,j})^{\eta}\right)^{-\sigma}
\end{aligned}
$$

The $\mathcal{M}^e$ matrices are obtained using a product of vectors element by element. The product operator starts every $\mathcal{N}^K$ element in such a way that $\mathcal{M}^e$ has $a = 1, ..., (n_h)^{n_s}$ columns *i.e* a column for each nodes of the Hermite integration. To compute the integral one has to use the weighting functions that are solved numerically. We denote by ω the vector of dimension n_hx 1 of Hermite weights in one dimension. With multiple aggregate shocks, we have the following

tensor product base for the nodes:

$$\mathcal{W}_p = \mathcal{I}_{(n_h)^{p-1}} \otimes \omega \otimes \mathcal{I}_{(n_h)^{n_s-p}} \qquad \text{for } p = 1, ..., n_s$$

The numerical integration requires to make a product of weights (element by element). We use the Hadamard product (or Schur product) (label by $\circ$) for this purpose.

$$\mathcal{W} = \mathcal{W}_1 \circ \mathcal{W}_2 \circ ... \circ \mathcal{W}_{n_s}$$

For convenience again, we reshape the vectors in matrices of dimensions $\mathcal{N}^K \text{x} \, (n_h)^{n_s}$. This will allow the combination of expectation values and weights.

$$\Omega = \mathcal{I}_{\mathcal{N}^K} \otimes \mathcal{W}^T$$

Where T stands for the transpose. Finally the expectations are obtained by multiplying the weights Ω and the expectations element by element.

$$\mathcal{E}^e \quad = \quad \left(\frac{1}{\sqrt{\pi}}\right)^{n_s} \circ \mathcal{M}^e \circ \Omega$$

Each term is multiplied by $\left(\frac{1}{\sqrt{\pi}}\right)^{n_s}$ to scale the weights since, by definition, $\int_{-\infty}^{+\infty} \omega_i d_i = \sqrt{\pi}$. Now, $\mathcal{E}^e$ is a $\mathcal{N}^K \text{x} \, (n_h)^{n_s}$ matrix. Each column j corresponds to the value of the expectation at a particular node. When summed up over j we get the following vector of expectations:

$$E_t \text{Y}^e_{t+1} \quad = \quad \sum_{j=1}^{n_h^{n_s}} \mathcal{E}^e_j$$

Finally, to parameterize the expectations we can solve for the coefficient Λ_E using ordinary least square as follow:

$$\Lambda_E = \mathcal{H}_t^T \mathcal{H}_t \left(\mathcal{H}_t^T E_t \text{Y}_{t+1}\right)$$

F.6 Newton algorithm

Λ_E are determined by ordinary least square. However, Λ_P can not be determined in a similar way because the model is highly non-linear. Even with a given vector of $E_t \text{Y}^e_{t+1}$ and the grid of the state variables $\mathcal{H}_t$ it is impossible to pin down the policy rules $\Phi(\mathcal{H}_t)$. Then, we use a Newton algorithm. It determines the coefficients of the policy rules that minimize the residuals in all equilibrium equations. We defined by $\mathcal{R}_{un}\left(\mathcal{H}_t, \Phi_{un}(\mathcal{H}_t), \Psi(\mathcal{H}_t)\right)$ and

$\mathcal{R}_{un}\left(\mathcal{H}_t, \Phi_c(\mathcal{H}_t), \Psi(\mathcal{H}_t)\right)$ the residuals of forward-looking equations in the unconstrained and constrained case respectively. The Newton algorithm[16] consists in finding the coefficients Λ_P that minimize the residual equations:

$$\Lambda_P^g = arg \min_{\Lambda_P^g} \mathcal{R}_g\left(\mathcal{H}_t, \Phi_g(\mathcal{H}_t), \Psi(\mathcal{H}_t)\right)$$

F.7 General algorithm

Step 1 *Choose the order of the Chebyshev polynomial D and the number of nodes $\mathcal{N}$ (which are at least equal to $D+1$). Build the multidimensional Chebyshev polynomials using the following recursion:*

$$T_n(x) = \cos(n \arccos(x))$$

Step 2 *Compute the grid of the 4 state variables, imposing the steady states to be equidistant from the upper bound and the lower bound of the grid. Use the Kronecker product to get the tensor product base $\mathcal{H}_t$.*

Step 3 *Initialize the policy rules coefficients Λ_P^{un} and Λ_P^c and the expectations function coefficients Λ_E. In our case we only set the coefficients associated to the constants to be equal to their deterministic steady state[17]. The rest being equal to zero. At this step, you should have an initial Guess for $\Phi_{un}(\mathcal{H}_t)$, $\Phi_c(\mathcal{H}_t)$ and $\Psi^e(\mathcal{H}_t)$*

Step 4: Expectations
a) Given $\Phi_{un}(\mathcal{H}_t)$, $\Phi_c(\mathcal{H}_t)$, use the Gauss-Hermite quadratures in order to compute the next period tensor grid $\mathcal{H}_{t+1}$.
b) Compute the policy rules $\Phi_{un}(\mathcal{H}_{t+1})$, $\Phi_c(\mathcal{H}_{t+1})$.
c) Compute the expectations $\Psi^e(\mathcal{H}_t)$ and use the ordinary least square to pin down the coefficients Λ_E.

Step 5 *Given $\Psi^e(\mathcal{H}_t)$, determine the policy rules coefficient Λ_P^g, $g = un, c$ using a Newton algorithm which minimize the residuals $\mathcal{R}_g\left(\mathcal{H}_t, \Phi_g(\mathcal{H}_t), \Psi(\mathcal{H}_t)\right)$ until reaching a criteria (10^{-11} in our algorithm). Pin down the policy rule $\Phi_g(\mathcal{H}_t)$.*

Step 6 *Check if the expectation functions are the same as in step 5 using an Euclidian norm and a convergence criterion of (10^{-8} in our algorithm). Formally it writes*

$$\frac{|\Lambda_E' - \Lambda_E|}{|\Lambda_E|} \leq 10^{-8}$$

Otherwise, return to step 4. Repeat this procedure until convergence.

[16] For this step we use a vectorial Jacobian computed numerically.

[17] It is possible to use a log-linear or perturbation method to initialize the coefficients. But the algorithm converges anyway.

G Accuracy of solution algorithm

G.1 Coverage of the grid points

One source of inaccuracy arises from the coverage of the tensor grid points. If simulated points are fairly outside the coverage implied by the tensor product basis, the model is not solved on points that are visited in equilibrium. This may involve potential spurious approximations. For this purpose we check if the ergodic distributions of states variables are *most of* included in the tensor product basis. By *most of*, we mean that it covers at least 95% (which is the case in our simulations $\simeq$ 99%). We start our algorithm with a tight grid and spread it to cover the simulated points. For the discount factor shock the minimum and maximum value of the grid points are about 3.5 times $\pm\frac{\sigma_\beta}{\sqrt{1-\rho_\beta^2}}$. For the unemployment benefits shock it is set to 10 times $\frac{\pm\sigma_\tau}{\sqrt{1-\rho_\tau^2}}$ to cover the estimated unemployment benefits shock that proxy the benefits extension.

G.2 Residuals in equilibrium equations

Den Haan & Marcet (1990), Den Haan & Marcet (1994) and Judd (1998) use a simple and powerful algorithm to evaluate the accuracy of dynamic models. They compute the residual of the Euler equation using Gaus-Hermite quadratures over the simulated series. We perform a similar exercise and find that the norm of all residuals are below 10^-4 on average, which is highly acceptable.

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This research was supported by the Deutsche Forschungsgemeinschaft through the SFB 649 "Economic Risk".

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This research was supported by the Deutsche Forschungsgemeinschaft through the SFB 649 "Economic Risk".