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# Corporate Cash Hoarding in a Model with Liquidity Constraints

Falk Mazelis\*



\* Humboldt-Universität zu Berlin, Germany

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# Corporate Cash Hoarding in a Model with Liquidity Constraints\*

Falk Mazelis<sup>†</sup>

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## Abstract

This paper studies the role of uncertainty in the corporate cash hoarding puzzle. The baseline model is a stochastic neoclassical growth model featuring idiosyncratic and uninsurable productivity shocks and a cash-in-advance constraint on new investments on the individual firm level. Individual agents' choices regarding cash holdings are analyzed. After a wealth threshold is reached, the cash-in-advance constraint ceases to have an effect on the agent's behavior. The resulting aggregate cash holdings of households increases with uncertainty. Aggregate consumption is also higher, but the added volatility of consumption decreases lifetime utility. Allowing firms to borrow and lend available unused cash increases average variables. An exogenous increase in the interest rate at which they intermediate funds leads to increased intermediation activity, corresponding to the lending channel of monetary policy transmission.

**Keywords:** Cash-in-advance constraint, idiosyncratic risk, corporate cash hoarding, lending channel

**JEL Classification:** C63, E21, E41, D81

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<sup>†</sup>falkmazelis@gmail.com, Research Training Group "Interdependencies in the Regulation of Markets", Humboldt University Berlin, Spandauer Str. 1, 10178 Berlin

# 1 Introduction

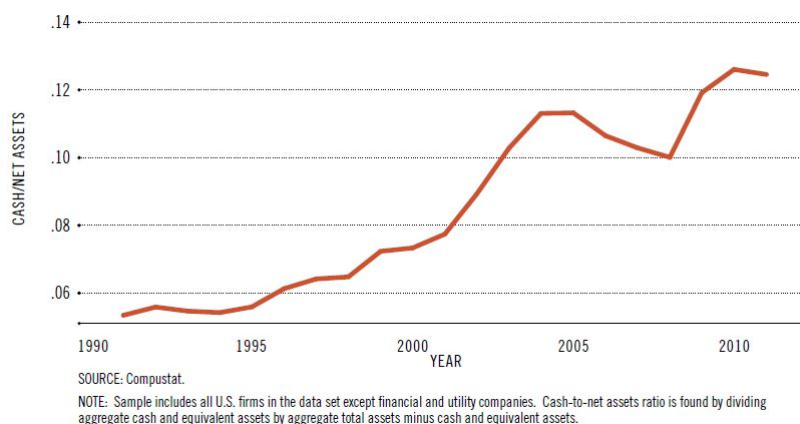


Figure 1: Ratio of Cash to Net Assets. Source: Sachnez and Yurdugal, 2012.

The ratio of cash and short term investments to total assets of publicly traded non-financial non-utility US corporations has on average increased from less than 6% in 1990 to more than 12% in 2011 (Bates, Kahle and Stulz, 2009), see Figure 1. There are many reasons for this, ranging from tax reasons to precautionary savings (Sanchez and Yurdugal, 2012). The precautionary motive gains additional weight when considering the answers to a survey of Chief Financial Officers of corporations globally, who state cash flow shortfalls and uncertainty about future investment opportunities among the most relevant reasons for excess cash holdings (Lins, Servaes and Tufano, 2010). In the same survey, the CFOs report that internal cash balances can be cheaper than accessing outside sources of funds. Furthermore, McVanel and Perevalov (2008) find a steady rise in the cash flow variability of Canadian firms from the 1980s to the mid-2000s, a development that tracks the rise of excess cash on company balance sheets. These findings indicate that uncertain environments may be partly responsible for increases in cash hoarding.

Although firms are commonly modeled as maximizing their expected lifetime profits by choosing the input factors of production optimally (see, for example, Woodford, 2002), these investments are often not constrained by any liquidity considerations of the firm, which implies perfect trust in the future settlement of suppliers' advance. In reality, however, firms carefully manage their cash holdings (and capital structure) to insure adequate cash holdings at every point: liquidity holdings below a certain level will prevent firms from meeting ongoing expenses and, possibly worse, prohibit profitable investments from being undertaken (unless the firm engages in restrictively expensive last minute lines of credit). Too high a cash balance is problematic in its own terms, since the funds could be either spent more profitably by investing in assets necessary for generating future revenue or paid out as dividends to the owners of the firm.

This model studies an agent that chooses her cash balance to balance her motive

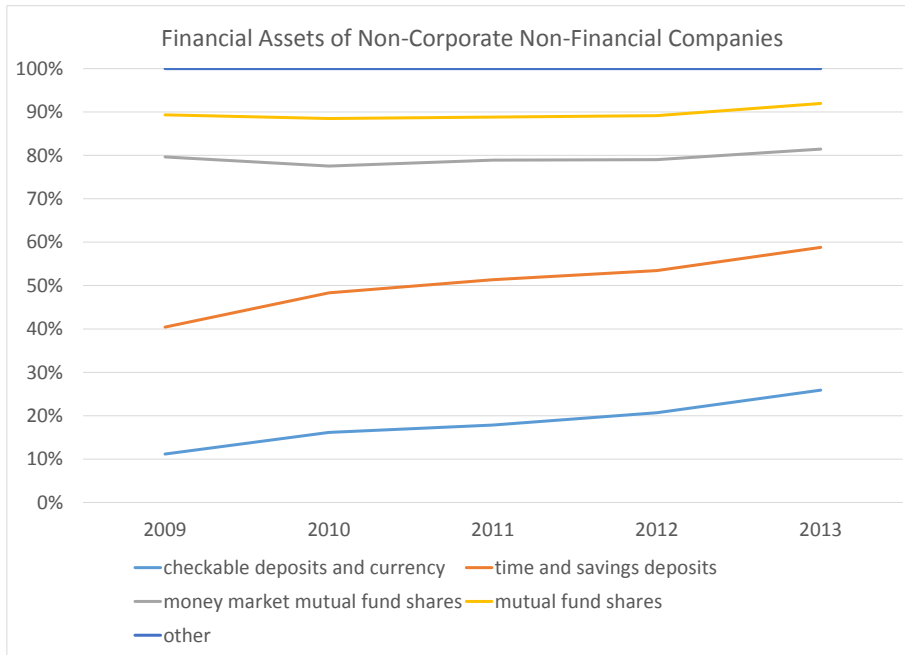


Figure 2: Distribution of Financial Assets, NCNF Companies. Source: US Flow of Funds, 2014.

of profitability with her responsiveness to productivity shocks. The circumstance of a prohibitive minimum cash balance is modeled as a Clower, or cash-in-advance (CIA), constraint. The constraining variable is chosen in the preceding period and this establishes an upper bound for certain expenses, which may be binding, depending on the current choice of those expenses. Establishing a value for this upper bound, which is decided endogenously, is simple in the case of certainty, because it merely has to cover predetermined expenses. In the case of uncertainty, however, the optimal upper bound depends on the expectations of expenses, which depend on the realization of the productivity shock. There is no insurance mechanism apart from self insurance through increased wealth holdings. The non-linear nature of the CIA constraint rules out perturbation techniques. Also, since there is only one law of motion for two state variables, I need to define a grid of possible values for one of those state variables to find a solution. This analysis therefore uses value function iteration to solve the model. I receive a policy function for cash holdings that varies with the amount of uncertainty, increasing excess holdings in more risky environments. In addition, I study the effects of one firm lending available cash to another firm, and the change that an interest rate increase brings about.

Cash hoarding is not a problem per se, since cash and short term investments are usually held by financial intermediaries, who will channel the funds to other likely profitable ventures. Non-corporate non-financial US companies keep their financial assets partly in banks and partly in non-banks, like money market mutual funds, see Figure

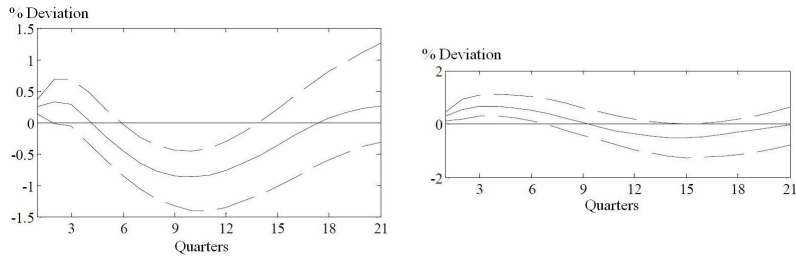


Figure 3: Response to Lending by Banks and Money Market Mutual Funds after a 1% increase in the Federal Funds Rate. Source: Igan et al, 2013.

2. This allocation of funds increases the interconnectivity between the real and financial sector, however, and may propagate financial risk to the real economy (Gertler and Kiyotaki, 2010). In addition, since monetary policy affects the real economy through the financial sector, different degrees of connectivity may have varying effects on the real economy. Bank lending and money market mutual fund lending react in different directions following an increase in the monetary policy rate, see Figure 3.

The paper is structured as follows. In section 2 the individual agent’s problem is presented. The steady state and dynamics are discussed. Section 3 explains the choice of parameters and explains some computational issues. Section 4 presents the global policy functions and aggregates individual decisions to a full distribution of agents. The effects of the CIA constraint and the effects of idiosyncratic risk on aggregate variables are decomposed, giving rise to two different kinds of precautionary savings. A financial sector is introduced in Section 5 that intermediates between cash rich and cash poor households. In a simulation, we show how exogenous interest rate increases can lead to an increase in the funds channeled through the intermediary. The last section concludes.

## 2 The Individual Agent’s Problem

In order to study the role of uncertainty on corporate cash balance sheets, we will build a model of a household that operates a firm. The household behaves like the manager of a firm who needs to invest into capital for the production process to produce consumption goods, which provide her with utility. In order to invest into the production process, she needs to hold a liquid balance to finance the capital expenditure. The connection with the real world is that companies have to continually pay dividends (consumption in this model) that they receive through their production processes. Investing into the production process takes time, which is why capital only becomes productive with a one period lag. Physical capital depreciates and has to be replaced. The replacement is done through investment, which is costly. The investment can ideally be financed through equity or debt issuance. However, profitable investment opportunities often fade quickly and changes to the capital structure can take a long time and are costly (see Bates, Kahlte and Stulz, 2009). Once the capital is in place, the productivity of the production

process can vary. Depending on the realization of productivity, the household is left with more or less resources for either consumption or further investment. The uncertainty about the production process therefore drives the cash balance of the firm.

## 2.1 The infinitely lived household

Consider an infinitely lived household that maximizes discounted utility, which is strictly increasing and strictly concave in consumption:

$$\max_{c_t, k_t, D_t} E_0 \sum_{t=0}^{\infty} \beta^t U(c_t) \quad (1)$$

The household's behavior is subject to a budget constraint and a CIA constraint:

$$c_t + k_t + D_t = A_t f(k_{t-1}) + (1 - \delta)k_{t-1} + D_{t-1} \quad \{\lambda_t\} \quad (2)$$

$$k_t \leq D_{t-1} + (1 - \delta)k_{t-1} \quad \{\mu_t\} \quad (3)$$

$$A_t = \text{stochastic process} \quad (4)$$

where  $c$  is consumption,  $k$  is the capital stock,  $D$  is deposits at a bank which we will treat as equivalent to cash,  $A$  is the level of technology,  $f$  is the production function,  $\delta$  is the depreciation rate on capital.

The household can either consume or choose to invest into capital or deposit holdings. By investing into capital, the household receives a return from production as well as undepreciated capital the following period. In addition, at the end of the current period the agent can carry over any wealth into the next period by depositing it at a bank at no interest. A CIA constraint motivates the agent to do this: Every period, the maximal amount of investments into capital can not exceed the amount of deposit holdings at the bank. The intuition for this is that the household needs a basket of different goods for investment that it does not itself produce. This basket contains goods that may not necessarily be traded for the household's own produce this period, but instead can be bought using a liquid asset in the form of bank deposits. The household can always sell all of its produce on the market and receive deposits in return. Note that deposits are a real good, which can also be consumed in case it is not used for investment purposes.

Technology  $A$  underlies a stochastic process. The revenue generated from production can therefore not be determined in advance. There are two interpretations for this. In the case of a persistent shock process, technology can be taken as profitable investments. This corresponds to an acquisition of one business by another. However, as in Hopenhayn and Rogerson (1993), this can also be interpreted as a consumer taste shock that shifts the demand curve. This is also true in case of no persistence.

Letting  $\lambda, \mu$  be the Kuhn-Tucker multipliers on the constraints, the resulting First-order conditions for consumption, capital and deposits are, respectively <sup>1</sup>,

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<sup>1</sup>The Lagrangian is

$$L = u(c_t) - \lambda_t [c_t + k_t - (1 - \delta)k_{t-1} + D_t - A_t f(k_{t-1}) - D_{t-1}] - \mu_t [k_t - (1 - \delta)k_{t-1} - D_{t-1}] \\ + \beta \{u(c_{t+1}) - \lambda_{t+1} [c_{t+1} + k_{t+1} - (1 - \delta)k_t + D_{t+1} - A_{t+1} f(k_t) - D_t] - \mu_{t+1} [k_{t+1} - (1 - \delta)k_t - D_t]\}$$

$$U'(c_t) = \lambda_t \quad (5)$$

$$\lambda_t + \mu_t = \beta E_t \{ \lambda_{t+1} [A_{t+1} f'(k_t) + 1 - \delta] + \mu_{t+1} (1 - \delta) \} \quad (6)$$

$$\lambda_t = \beta E_t \{ \lambda_{t+1} + \mu_{t+1} \}. \quad (7)$$

$$[D_{t-1} - k_t + (1 - \delta)k_{t-1}] \mu_t = 0, \quad D_{t-1} - k_t + (1 - \delta)k_{t-1} \geq 0, \quad \mu_t \geq 0 \quad (8)$$

Equation 5 states that the marginal utility of consumption equals the marginal utility of wealth  $\lambda_t$ . Equation 7 states that capital should be invested up until the combined cost of having an additional unit of wealth  $\lambda_t$  and at the same time an additional unit of liquidity bearing assets  $\mu_t$ , which is made up of deposits and undepreciated capital from the previous period, has to be equal to the combined value of the marginal capital return, which is the marginal product from production including next-period's undepreciated capital, weighted by tomorrow's marginal utility plus the future liquidity services the undepreciated future capital will deliver. Equation 7 states that the cost of investing a unit of wealth into deposits today has to equal the value of having that additional unit deliver wealth or liquidity services tomorrow.

## 2.2 The Deterministic Steady State

In the deterministic steady state, the stochastic process regarding technology is turned off and the steady state values for capital, consumption and deposits can be solved from, respectively,

$$\begin{aligned} f'(k) &= \frac{1}{\beta} \left( \frac{1}{\beta} + \delta - 1 \right) \\ c &= f(k) - \delta k \\ D &= \delta k. \end{aligned}$$

Compared to the neoclassical growth model without a CIA constraint, the marginal product of capital is higher by  $\frac{1}{\beta}$ , because the additional liquidity services from cash are costly and have to be financed one period in advance as can be seen in the three period model, where production in period 3 depended on the choice of deposit holdings in period 1.

The CIA constraint is binding, since the first order condition with respect to deposits (7) would otherwise lead to a contradiction: a non-binding CIA constraint implies  $\mu = 0$ , which results in  $\beta = 1$ . This means that the household holds a deposit balance equal to the replacement investment each period due to depreciation. Since the marginal product of capital dominates the marginal product of cash, which does not carry any interest, additional deposit holdings are never profitable. A smaller deposit balance would prevent the household from maintaining its steady state capital level.

### 2.3 Dynamics in the Infinite Time Model

Adding stochastics changes the rational regarding asset holdings. The optimal asset level rises with uncertainty. Assets in the model are comprised of capital and deposits. Interpolating from the two period to the infinite period optimization, the amount of capital holdings increases with risk.

Additionally, the CIA constraint introduces a further motive for precautionary savings as not to hit the liquidity constraint. The household will always prefer to invest into capital, as long as the marginal productivity of capital is above the marginal productivity of deposits, which does not carry any interest and is always 0. However, a positive shock may induce the household to save more deposits as to realize higher capital investments in the following period. Optimal deposit holdings depend on the exact nature of the shock.

The additional CIA constraint changes the household's precautionary savings motive. Not only does self-insurance against adverse shocks play a role. In addition, hitting the CIA constraint is costly in utility terms as well. The household may have an incentive to insure against hitting the liquidity constraint, too.

When no uncertainty is present and the inequality constraint in the model description always binds we can determine deposit holdings each period exactly from the first order condition for next period's optimal capital, which together with our current period capital gives us our deposit holdings exactly. When we do not know whether the inequality constraint is binding or not, because next period's optimal capital can not yet be determined, since the shock has not realized, finding the optimal value for deposit holdings has to be done numerically.

## 3 Model Specification, Parametrization, and Computation

In this section, we will specify the model further and choose parameter values for solving the model. The computational method and the kind of aggregation is discussed.

Household utility is of isoelastic utility form  $U(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$  with a coefficient of relative risk aversion  $\gamma$ . The production function is assumed to be concave, i.e.  $f(k) = k^\alpha$  and specified by output elasticity  $\alpha$ . The stochastic process in this model is  $A_t = \bar{A}e^{\rho \log A_{t-1} + \varepsilon_t}$ , where  $\bar{A}$  is the steady state value of technology and  $\varepsilon$  is an idiosyncratic shock to technology that follows a white noise process,  $\varepsilon \sim N(0, \sigma^2)$ .

The parameter values are mostly taken from Aiyagari (1994), who specifies the model period as one year. The utility discount factor  $\beta$  is 0.96. The output elasticity  $\alpha$  equals 0.36. Depreciation  $\delta$  is 0.08. As a coefficient of relative risk aversion  $\gamma$ , I choose 3 out of the ones offered by Aiyagari.

For the shock process, I am choosing no autocorrelation for computational reasons, explained further below. Choosing a standard deviation for the shock process is tricky. In the literature, there are various explanations for standard deviations for idiosyncratic shocks to labor and for aggregate technology shocks. It is harder to find a meaningful parameter for idiosyncratic shocks to a firm's technology in a simple framework like



this. Pesaran and Xu (2011) estimate a  $\sigma = .041$  and Covas (2005) chooses  $\sigma = 0.4$ , which is in line with McVanel and Perevalov (2008) when fluctuations in technology are interpreted as variations in cash flow. Covas has a similarly basic model with a borrowing constraint albeit without a CIA constraint. He performs a sensitivity analysis by varying the standard deviation and persistence and finds a sizeable difference between aggregate capital in a model with uninsurable productivity risk and complete markets depending on the parametrization. With a CIA constraint in place, this effect is likely to vary since not only is there a borrowing constraint in place but an additional constraint on another dimension. I choose  $\sigma = 0.4$  and will perform a sensitivity analysis below.

### *Computation*

The computational method is a value function iteration that was performed using Matlab. The algorithm is standard and I only highlight some parameter values and one divergence from my own model.

Although the model is specified in terms of capital and deposits, both variables play the role of savings devices. Instead of evaluating the model at absolute values of these variables, I have rewritten the model such that the first variable is combined wealth, equal to capital and deposits, and the second variable is the percentage of wealth that is held in deposits.

$$k_t + D_t \equiv x_t$$

$$\frac{D_t}{k_t + D_t} \equiv d_t$$

This is done for computational reasons: In VFI it is a problem when the derived policy functions hit the grid boundaries since I can not be sure whether the agent would have preferred a value outside the grid. The grid is usually chosen in such a way that most of the dynamics take place towards the middle. When choosing absolute values for both variables, the method evaluates extreme areas as well: those with low and high values for both variables at the grid end points. Since an agent with a high level of combined capital and deposits would choose a very high capital value that is not within the grid bounds, this may distort the policy functions, although an agent would never visit these areas given a steady state and a stochastic process. Including this higher capital value on a successive larger grid exaggerates the problem, since the reaction to this grid point including deposits is yet higher. For absolute values there is no upper bound. Instead, the absolute variable wealth can be distributed optimally by a relative variable that is bound between 0% and 100% and if these grid end point are reached, they are a naturally occurring extreme.

The shock process is modeled using Tauchen's (1986) method.

### *Aggregation*

The economy is populated with a continuum of households all of the type discussed in Section 2. Denoting individual variables with  $j \in [0,1]$ , we can now write aggregate

variables as

$$\begin{aligned}
c^A &= \int_0^1 c^j dj \\
k^A &= \int_0^1 k^j dj \\
D^A &= \int_0^1 D^j dj \\
y^A &= \int_0^1 (k^j)^\alpha dj
\end{aligned}$$

Importantly, aggregate production consist of adding all the individual production functions, instead of assuming aggregate capital as the input in the production function.

To compute the stationary distribution, we follow the approach by Maussner and Heer (2005) using a Monte Carlo simulation. Every period, the household's technology is hit with a shock and the household reacts to it depending on its state. Since the grids are discrete, eventually all possible state combinations are visited by the agent. Equivalently, we can look at a distribution of agents, starting with the total mass concentrated on one state, and look at the reaction to each possible shock. Since the shock realizations are on a discrete grid, the first such reaction will be control vectors for wealth and deposit percentage of a size equal to or less than the number of shock realizations. Each of these control vectors can be regarded as possible states in the following period and again hit with all possible realizations of the shock. Since all grids are discretized, the maximal combination of states and shocks is the number of grid points of both state variables multiplied with each other and with the number of realizations of the idiosyncratic shock. Eventually, if further shocks only change individual agents' reaction but not the distribution across state combinations, we arrive at a stationary distribution. Fortunately, this is the case for this model<sup>2</sup>. This stationary distribution from idiosyncratic shocks can then be analyzed in comparison to the complete markets case.

Since the grids in this model are discretized, I am receiving a stationary distribution that has mass concentrated on specific state combinations. Since the discretization is only a computational tool to solve the policy functions and the policy functions would themselves be continuous, I will present the resulting stationary distribution after fitting a probability distribution to the data. I am using the MATLAB command `fitdist` with a normal kernel smoother along both dimensions and a window size of 1 for th wealth dimension and a window size of 0.05 for deposit percentage.

Aggregation for comparing the results is done using the discretized results. Since the distribution is stationary, I simply aggregate the state variables capital and deposits with regard to their mass. For consumption the realization of the shock matters. I therefore aggregate over their mass conditional on the shock they receive.

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<sup>2</sup>I have yet to prove whether this is the case in general and for which parameter space

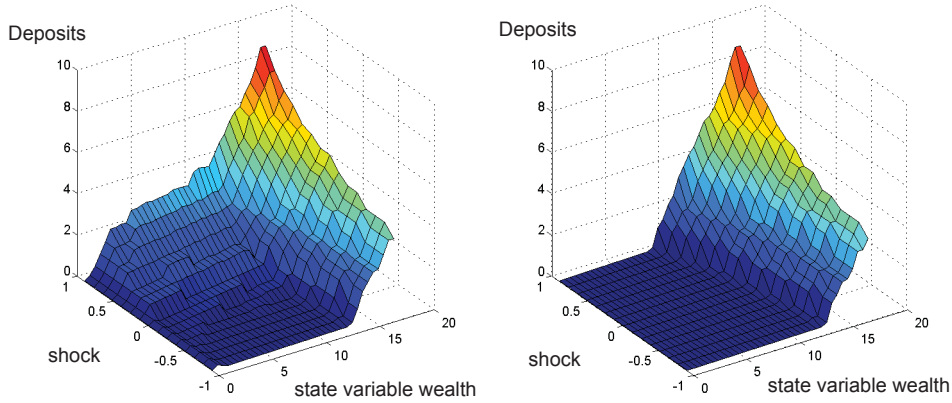


Figure 4: Policy function for Deposits, constrained (left) and unconstrained

$$\begin{aligned}
 c^A &= \sum_0^1 c^j dj \\
 k^A &= \int_0^1 k^j dj \\
 D^A &= \int_0^1 D^j dj \\
 y^A &= \int_0^1 (k^j)^\alpha dj
 \end{aligned}$$

It is important to note that every agent produces using their own capital holdings as input for the production function. Therefore each agent will have a different marginal productivity, depending both on individual capital holdings and idiosyncratic technology shock.

## 4 Results

### 4.1 Policy Functions

The household reacts to the current technology shock  $\varepsilon_t$  and state variables  $x_{t-1}$  and  $d_{t-1}$  by choosing current consumption  $c_t$ , current wealth  $x_t$  and percentage invested in deposits  $d_t$ . Although all policy functions are highly non-linear, the key result is the difference in the policy function for deposits in the case of a constrained vs an unconstrained agent.

For wealthy firms, the value of additional consumption and additional production does not exceed the value of additional deposits anymore. As can be seen from figure 4,

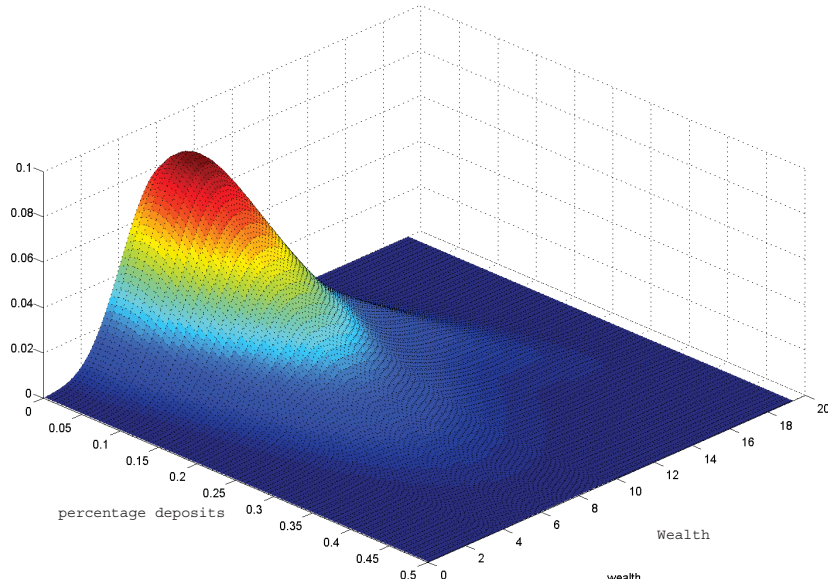


Figure 5: Stationary distribution over states wealth and deposit percentage

the behavior of a constrained wealthy firm is identical to the behavior of an unconstrained wealthy firm. Wealthy firms are those that own state variable wealth of more than 12. Once wealth exceeds a boundary, the CIA constraint does not play any role anymore. In both cases, investments into deposits increase with the size of the shock and the amount of wealth.

Wealthy firms value the certain return of cash over the uncertain but possibly higher return of additional production. For less wealthy firms, the policy function for deposits increases mostly in the size of the shock: higher available resources lead to additional investments into deposits because the firm insures against hitting the CIA constraint. This is clearly not a motivation in the unconstrained case: all available resources will be either consumed or invested into production, because the marginal product of capital exceeds the marginal product of deposits, and uncertain returns are more valuable than uncertain ones.

## 4.2 Aggregation

The heterogeneous agent economy converges to a stationary distribution after some time. As can be seen from figure 5 the distribution of agents is spread across the wealth and deposit percentage dimensions. Two things stand out. First, the distribution is bound by zero deposit holdings to the north west of the figure. Since agents can not borrow, the distribution can not spread out across the zero threshold. Without a borrowing constraint we might be able to see the distribution extend across the threshold.

This is a problem for many households at the zero deposit threshold because zero

deposit holdings mean that they can not make any net new capital expenditures beyond their undepreciated capital. Even if a favorable shock occurs, they will need to allocate the additional return from production to deposits first and can only invest into capital the following period. The difference between the possible marginal return on capital and the marginal return on deposits is a loss to the household that can not be regained.

At the same time, there are many agents that do not need all their deposit holdings, because their realized investment is smaller. By investing into deposits they are insuring against the case that they are going to be constrained by the CIA constraint and lose the difference between marginal productivity of capital and deposits themselves. All agents could therefore benefit by allocating some of the deposits of the deposit-rich agents to the deposit-poor agents that would like to invest. The additional marginal productivity could be split in some way that makes all agents better off. Such a financial intermediary does not exist in this model.

The second observation is the distribution of agents that is continually hit with positive shocks and therefore keeps accumulating wealth. It is visible in figure 5 as the mass branching off from the probability distribution to the north east. Wealth holdings increase and the agents allocate more of their wealth into deposits. These are the agents that are holding wealth higher than 12 in this model. Their value from liquidity services is greater than their marginal productivity of additional capital or the additional marginal utility from consumption, and this value increases quickly, inducing ever higher deposit holdings.

This fact can be observed well in figure 6, a Lorenz curve for deposit holdings. The distribution of deposit holdings is distributed very unequally across the population. A financial intermediary could allocate this liquidity from deposit-rich to deposit-poor households to meet the CIA constraint.

### 4.3 Decomposition into Liquidity and Risk Motives

We have introduced two mechanisms into the model that cause precautionary savings of two different types. One is the well known precautionary motive for wealth holdings to insure against idiosyncratic risk. The other one is precautionary savings against hitting the liquidity constraint and not being able to invest optimally into capital. I will call these two different precautionary motives PS1 and PS2 after Xu (1995) who studied precautionary savings in the income fluctuation problem from idiosyncratic risk and a borrowing constraint. By solving a model with and without CIA constraint (but borrowing constraints) I can compare the values of the aggregate economy with their steady state counterparts. Precautionary savings are then defined as:

$$PS1 = wealth^{noCIA} - wealth^{SS}$$

$$PS2 = wealth^{CIA} - wealth^{noCIA}$$

Table 1 compares aggregate values from the stationary distribution with their steady state counterparts in a model of idiosyncratic risk but no CIA constraint. Agents insure

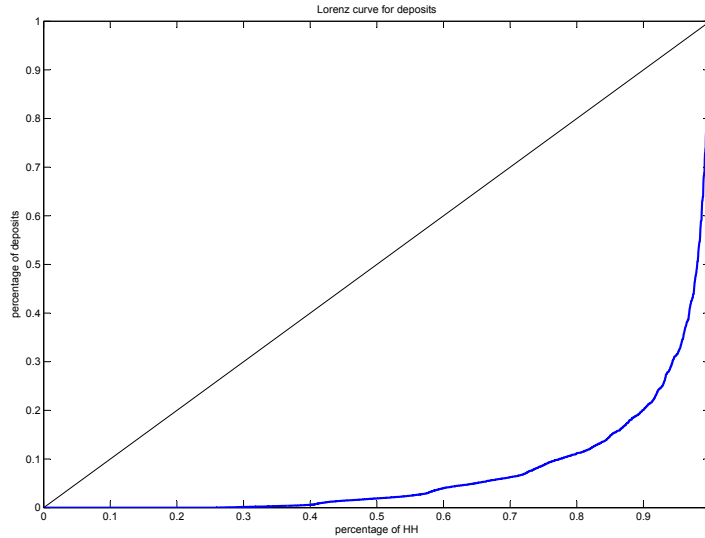


Figure 6: Lorenz curve for deposits

	Consumption	Capital	Deposits	Wealth
Steady State	1.39	5.11	0	5.11
Idiosyncratic Risk	3.2	14.16	0.21	14.37
Deviation	230%	277%	NA	281%

Table 1: Comparison between steady state and aggregate values without CIA constraint

mostly by accumulating more capital up until the point at which the marginal productivity of capital becomes less than the marginal productivity of deposits. This is indeed the case in this model under our current parameterization. The aggregate values for the economy deviate significantly from their steady state values. This deviation is obviously related to the standard deviation of the shock.

Table 2 compares steady state values of a model with idiosyncratic risk and a CIA constraint with their aggregate realization. The difference in the steady states is only in deposits, since the agent in the CIA constraint model needs deposits each period that cover the investment expenses for depreciated capital. The aggregate realizations are even higher, which is unsurprising given that the households have to insure along two dimensions. Especially of interest is the fact that firms hoard enormous amounts of the liquid asset without even planning on spending it. One motive is the uncertainty about investment opportunities. Secondly, however, since uncertainty already increases their capital holdings, firms' marginal productivity of capital is further reduced. This reduction brings the low marginal productivity of deposits less unattractive. The result is a further investment into these assets. Excess cash hoarding is the result.

	Consumption	Capital	Deposits	Wealth
Steady State	1.39	5.11	0.41	5.52
CIA constraint	6.36	26.76	2.79	29.55
Deviation	458%	524%	680%	535%

Table 2: Comparison between steady state and aggregate values with CIA constraint

CRRA	Consumption	Capital	Deposits	Wealth
Steady State	1.39	5.11	0.41	5.52
0.5	1.57	5.91	0.55	6.46
1	1.57	6.04	0.58	6.62
2	3.16	12.7	1.26	13.95
3	6.36	26.76	2.79	29.55

Table 3: Comparison of aggregate values with CIA constraint for different values of  $\gamma$

Curiously, aggregate consumption is higher in both models than in the steady state. This may be surprising, since the agents only value consumption in utility and a higher aggregate level in a stationary distribution would mean that risk and even a constraint increase utility. This is only true of the aggregate level, however. In time, agents in both models visit all of the different levels of consumption eventually. This fluctuation in consumption is costly: Utility of an (aggregate) agent living in the steady state, the idiosyncratic but unconstrained model, and the CIA constrained model are, respectively -0.26, -0.42 and -0.85. Agents would therefore always prefer the steady state to the idiosyncratic and to the constrained model.

#### 4.4 Sensitivity Analysis

In order to study how sensitive the model results are to parameter values, I am showing key results from different parameterizations. Most of the parameters are directly taken from Aiyagari (1994). However, no counter part from that paper exists for a technology shock. In addition, the economic interpretation of this shock process can differ. I therefore present aggregate variables of the economy for different values of the standard deviation as shown in Figure 7.

In addition, since this study is concerned with precautionary savings, a variation of the coefficient of relative risk aversion is in order. Table 3 shows values for aggregate variables of the economy for different values of the coefficients of relative risk aversion. The higher the coefficient of relative risk aversion the more negatively a household is affected by changing consumption. Because volatility in consumption is bad, a household will self insure against negative consumption outcomes by having a higher consumption level in expectations.

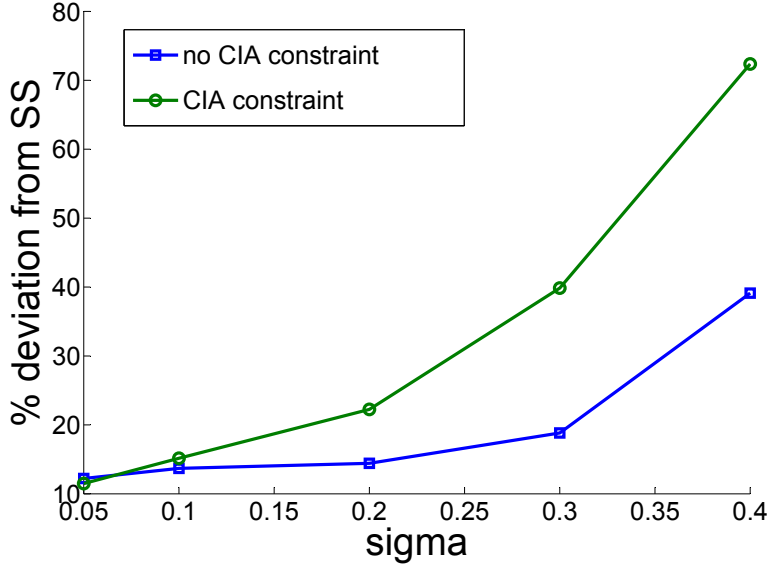


Figure 7: Deviation of deposits from steady state value for different standard deviation of the shock

## 5 Introducing a Financial Sector

As explained above, households need liquid assets to finance investments. These liquid assets take the form of deposits, which are simply created by an exogenously assumed financial sector. This sector holds the liquid assets for the households and trades them for investments when so demanded. In reality, the banks are themselves economic entities that maximize profits by finding profitable investment opportunities. Investment for them takes the form of loans into businesses that are cash constrained. This model features many agents that are cash constrained, namely those whose CIA constraint is binding. We will now discuss whether and how the financial sector can distribute funds from households with available funds to households with demand for available funds, and how the interest rate affects lending.

For this experiment we need to allow the household to borrow and lend funds. The household's budget constraint and CIA constraint are modified accordingly:

$$c_t + k_t + D_t = A_t f(k_{t-1}) + (1 - \delta)k_{t-1} + D_{t-1} + r_t F_t \quad (9)$$

$$k_t \leq D_{t-1} + (1 - \delta)k_{t-1} - F_t. \quad (10)$$

$$(11)$$

The household has the possibility to lend out funds  $F_t$  at the exogenously given interest rate  $r_t$ , which increases the available resources (the right hand side of the bud-



get constraint) to invest into consumption, capital and deposits. The downside is that the CIA constraint becomes tighter. Depending on the amount of available funds, the household may prefer to lend or even borrow: negative values of funds  $F_t$  signify borrowing, which decreases the amount of available resources this period but softens the CIA constraint, thereby allowing more investment into capital. This makes sense if the cost of borrowing is less than the expected discounted additional marginal product of capital next period. Conditional on intermediation, the agent may pick investments in capital and deposits as well as consumption that are different from the case without intermediation.

We next assume that the only available funds for borrowing have to come from a household that wants to lend. We thus assume two firms that can borrow and lend from each other and that receive a certain value  $V(F_t, r_t)$  from intermediating funds  $F_t$  at interest rate  $r_t$ . In order for intermediation to take place, both firms have to prefer intermediation  $V^j(F_t, r_t)$  and  $V^{-j}(-F_t, r_t)$  to no intermediation  $V^j(0, .)$  and  $V^{-j}(0, .)$ . To verify this, we solve the value function iteration with two additional state variables, funds  $F_t$  and interest rate  $r_t$ , within certain bounds. For every combination of states, the VFI will evaluate all combinations of funds and interest rates. The resulting value function will indicate which funds/interest rate combinations are most valuable to the agent for each state. If one agent prefers to lend out funds at a given interest rate to not lending out any funds, and the other agent prefers borrowing that amount of funds at that interest rate to borrowing no funds, we have established a suitable intermediation match.

Often, there are several combinations of funds/interest rates that are preferable to both agents compared to the status quo of no lending. There are several ways of establishing which combination is selected. Apart from a random choice we can consider the maximum aggregate value of both agents' combinations, or the maximum value for either borrower or lender. The algorithm for selecting a fund/interest rate combination is then as follows.

*Algorithm for intermediation each period*

1. Draw a technology shock for each agent (correlated or uncorrelated).
2. Find values without intermediation for both agents  $V^j(0, .)$  and  $V^{-j}(0, .)$ .
3. Find values for all possible fund/interest rate combinations  $V^j(F_t, r_t)$  and  $V^{-j}(-F_t, r_t)$ .
4. Evaluate which combinations are preferable to both agents  $V^j(F_t, r_t) > V^j(0, .)$  and  $V^{-j}(-F_t, r_t) > V^{-j}(0, .)$ .
5. Pick combination based on a decision rule (random, lender/borrower/aggregate maximum).
6. If applicable, update choice for capital, deposits and consumption.

Depending on the decision rule, we receive different average values for our variables. Table 4 summarizes the results. Once we allow intermediation, the average values are not too different from each other. However, if we impose a different interest rate on intermediation from the one that was selected by the decision rule, we see different

Best Value	Shocks	Consumption	Capital	Deposits	Funds	Interest rate
No Intermediation		1.33	5.12	.57	0	-
Borrower	correlated	1.39	5.76	.63	.25	.02
	uncorrelated	1.38	5.77	.62	.27	.02
Lender	correlated	1.40	5.86	.64	.25	.025
	uncorrelated	1.4	5.98	.65	.27	.023
Aggregate	correlated	1.39	5.72	.63	.25	.02
	uncorrelated	1.42	6.12	.66	.27	.02

Table 4: Simulation results: average values for different types of fund intermediation

Best Value	Shocks	Intermediation	
		Increase (in %)	Decrease (in %)
Borrower	correlated	73	20
	uncorrelated	63	10
Lender	correlated	1	77
	uncorrelated	5	70
Aggregate	correlated	61	31
	uncorrelated	30	45

Table 5: Simulation results: average values for different types of fund intermediation

results for the amounts of funds intermediated based on decision rule and correlation.

For this analysis, we need to change the algorithm for intermediation to test for effects of exogenous interest rate increases. Specifically, after the algorithm has decided on a combination based on decision rule and correlation, we impose a higher interest rate and count the times that intermediation will go up, down, or stay unchanged. Table 5 shows the number of times that intermediated funds increased and decreased following an exogenous interest rate increase.

If the amount of intermediation is picked that delivers the highest value to the borrower, an exogenous increase in the interest rate will often lead to higher values for funds being intermediated. The intuition for this is clear: since we pick the optimal value for the borrower, the lender may face an inoptimal fund/interest rate combination. If the interest rate is increased despite the decision rule, the lender is induced to increase the amount of funds made available. Even though this fund/interest rate combination is suboptimal for the borrower (compared to her best value case), it is still better to borrow more at a higher interest rate and invest the additional funds into capital than to borrow less. This stands in contrast to the case when the best value for the lender is selected. An increase in the interest rate may induce the lender to make more funds available. However, the lender already receives their best value and an increase of the interest rate discourages the borrower from increasing demand for funds. The decision rule for highest aggregate value is an intermediate case: increases in the interest rate sometimes lead to more, sometimes to less intermediation.

## 6 Preliminary Conclusion

Firms faced with idiosyncratic shocks regarding their cash flows that cannot be fully insured against and that have to react to investment opportunities on the spot and without outside financing amass liquid holdings well in excess of the amount they actually are likely to spend in the next period.

Non wealthy firms invest into deposits in excess of their expected investment next period because an additional unit in deposits tomorrow will enable an additional investment into capital tomorrow. This is valuable, if the twice discounted marginal return on capital in two periods time is higher than the marginal return on capital tomorrow.

Wealthy firms invest into deposits because they value certain returns over risky returns. Importantly, wealthy firms' policy function for deposits is unaffected by a CIA constraint. The liquidity motive does not play a role.

Two additional mechanisms that drive higher cash holdings are at work. Firms increase their capital holdings because of their precautionary savings motive. This overinvestment into capital decreases average marginal productivity across the economy. Because the opportunity costs of deposits decline, the low marginal productivity of deposits becomes more attractive. Additionally, higher precautionary capital levels make higher replacement investments necessary. In order to finance these additional investments, precautionary deposits have to be held in advance.

Aggregate consumption in the case of uncertainty is higher than in the complete markets case. The reason for this is the high variability of productivity and the resulting volatility of marginal productivity. This volatile consumption, which is higher in the aggregate, is unfavorable to lifetime utility. Agents would prefer a less risky environment.

Households may benefit from redistributing deposits temporarily in order to allow deposit-poor households to reap high marginal productivity. Introducing a financial intermediary results in higher average values for consumption, capital and deposits. Depending on the decision rule, imposing higher interest rates exogenously has the effect of increasing intermediated funds, because lenders are more willing to make them available.

The increase in aggregate variables, partly attributable to the choice of parameter values, has effects on the general equilibrium. Once endogenous interest rates and prices are introduced, the absolute level of deposit holdings will influence these macro variables and vice versa.

The high level of liquid assets, which is usually managed by the financial sector, increases the connection between the real and the financial sector. Idiosyncratic firm risk has a direct effect on funding opportunities for banks and other types of financial intermediaries. Lending out hoarded cash can increase both borrower and lender utility.

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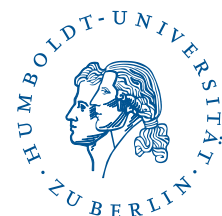
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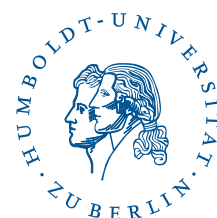
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