HEDGING ELECTRICITY PORTFOLIOS VIA STOCHASTIC PROGRAMMING

STEIN-ERIK FLETEN[†], STEIN W. WALLACE[‡], AND WILLIAM T. ZIEMBA[§]

Abstract. Electricity producers participating in the Nordic wholesale-level market face significant uncertainty in inflow to reservoirs and prices in the spot and contract markets. Taking the view of a single risk-averse producer, we propose a stochastic programming model for the coordination of physical generation resources with hedging through the forward and option market. Numerical results are presented for a five-stage, 256 scenario model that has a two year horizon.

Key words. Stochastic programming, hydro scheduling, portfolio management, deregulated electricity market.

1. Introduction. We discuss a portfolio model for a hydropower producer operating in a competitive electricity market. The portfolio includes one's own production and a set of power contracts for delivery or purchase, including contracts of financial nature. The advantages of using such a model compared to current industry practice is illustrated through an example.

Following deregulation, the producers in Scandinavia have had to change their focus from reliable and cost-efficient supply of electricity to more profit oriented and competitive objectives. Many countries are in the process of deregulating the electricity industry, often beginning at the wholesale level.

We assume the producer has access to functioning electricity forward and futures markets, providing derivative instruments for portfolio management. Such markets exist today in some countries, but are not ideal in terms of the number of available instruments and liquidity. Still, opportunities for diversification of risk, using electricity commodity markets, has made portfolio management techniques relevant for planning in the electric power industry. After deregulation, managers in electricity utilities are concerned with the large economic risks in their operation. These risks can

[*]Submitted to the IMA Volumes in Mathematics and its Applications, Springer Verlag.

[†]Department of Industrial Economics and Technology Management, Norwegian University of Science and Technology, Norway. E-mail: sef@iot.ntnu.no. The work of the first author was supported by the Norwegian Research Council (No. 111104/510) and members of the Norwegian Electricity Federation (No. 290104/400015). We have benefited from cooperation with Asbjørn Grundt and colleagues at Norsk Hydro Energy and Birger Mo at SINTEF Energy Research.

[‡]Department of Industrial Economics and Technology Management, Norwegian University of Science and Technology, Norway. E-mail: sww@iot.ntnu.no.

[§]Faculty of Commerce and Business Administration, University of British Columbia, Canada. E-mail: william.ziemba@commerce.ubc.ca.

be hedged in the contract market or reduced by adjusting operations decisions. To find the risk reduction decisions with the lowest cost in terms of reduction of expected value, we propose a portfolio management model that includes risk aversion, contract trading and electricity operating decisions.

The basic risk factors in this model include the wholesale spot price of electrical energy, derived contract prices and input price uncertainty. It is assumed that prices are unaffected by the decisions of the utility manager. Hence we are taking the perspective of a price taking electricity producer, operating in the wholesale market.

In the implementation of the model, the input factor uncertainty has the form of uncertain inflow to hydro reservoirs. Electricity generation is modeled at the level of detail common in medium to long term hydro planning models, without head variation effects. Contract types included are forwards and options. Thermal production is not included. Since the granularity of the model is one week at its finest, start- and shutdown costs are insignificant. Hence thermal production would be easy to include; it would not be necessary to use integer variables for modeling thermal generation.

The large transaction costs of contracts, including bid-ask spread, call for a dynamic contracting model. Thus one of the problems recognized by this model is the tradeoff involved in incurring transaction costs now, versus the cost/benefit of waiting for more information. Hydroelectric scheduling is also a dynamic problem, where the decision to release water now involves a tradeoff between reduced risk of spill and reduced risk of having to sell at low prices in the future. A stochastic programming approach is therefore appropriate to support the managing of both the "power portfolio" and the financial portfolio of hydropower producers. On the operations side of hydropower production, stochastic programming methods have been used in Sweden and Norway for many years, originating from the work of Stage and Larsson [15] and others.

We model the integrated portfolio selection and hydropower scheduling problem as a multistage linear stochastic program. The producers maximize expected profits subject to a risk constraint. Stochastic parameters are electricity prices, prices of financial instruments and inflows to reservoirs. Inflow uncertainty and price uncertainty are of particular importance for explaining varying financial performances for the producers. The industry is based on a mix of thermal power and hydropower, with the hydropower dependence making spot prices correlated with the amount of inflow to reservoirs. The last fact is due to the correlation between local and regional precipitation: water shortage or abundance is often national, not just local. Besides, much of the residential heating is done via electricity, so that if the temperature is very low, then not only is demand higher, but there is also likely to be less inflow.

We believe the model can provide the producer with a starting point in making decisions regarding power scheduling, contracting and coordination between those activities. It can provide important information regarding tradeoff between risk and expected return on short and long term, given the resources available. It is easy to use in conjunction with shorter-term power scheduling models, as it can provide risk-adjusted incremental values of stored water in reservoirs for the end of the first week. Our numerical example demonstrates some of these aspects.

For a survey of multistage portfolio models, specifically asset and liability management models, see [19]. We draw upon that literature when modeling contracts and risk aversion. Portfolio management models for energy firms are rare in the literature. A contract portfolio model for a gas producer is presented in [8]. Both static mean-variance and dynamic stochastic programming versions are explored. The aim was to find the optimal allocation of gas production capacity to different segments of contracts of the North-American gas market. The background of this work is a portfolio model that manages hydro production and future contracts in a competitive electricity market [5], and the further exploration of that model in [7].

In Section 2 we discuss relevant aspects of the electricity markets. Section 3 presents the model, Section 4 shows how the scenarios are generated and Section 5 discusses model validation. A numerical example is given in Section 6, implementation issues are covered in Section 7, possible further developments are discussed in Section 8 and Section 9 has concluding comments.

2. Electricity markets. The Nordic power exchange currently comprises Norway, Sweden, western Denmark and Finland. The transmission and generation services are unbundled, i.e. there is free access (common carriage) over the network. We discuss here the generation side of the business. In Norway, this unbundling is accomplished by regulating the transmission side and having a "free" market on the generation side of the industry. There is a legal requirement for power utilities to have separate financial reporting for transmission and generation. This is weaker than requiring splitting of companies, although this is the intention of the Norwegian Energy Act.

In the Nordic region there are two markets for electricity contracts excluding the shorter term spot markets: the Nord Pool organized markets and a bilateral market for over-the-counter (OTC) contracts. Today about 75% of the total turnover of derivative contracts is in the OTC market. In the OTC market the most common contract types are forward contracts with different (fixed) load profiles, options and forward contracts with flexibility in the load profile (load factor contracts). An important type is the flexible load factor contract. A typical load factor contract has a one-year maturity, 5000 hours of maximum load, with the additional constraint that 2/3 of the contract energy volume must be utilized in the summer season, and 1/3 in the winter season.

The markets organized by Nord Pool are classified according to the time scale of the contracts traded; there is a "regulating power" market, a spot market and a futures market. The regulating market is operated by Statnett, who has the technical responsibility for the main grid. It is used for matching real time supply and demand. Market participants with technical ability to rapidly control their power flow submit bids to Statnett on how they can ramp up or down at which price. Statnett chooses the most economical way to control the system according to the merit order list. The prices in this market are settled ex post to be equal to the bid price of the producer that was picked by Statnett.

What is termed the spot market is actually a forward market settled daily at noon for delivery in the next 12–36 hours. It is meant to reflect the marginal price under the prevailing conditions, and was based on the former power pool market established with restricted access in 1970. The individual supply and demand curves submitted by all participants are aggregated by Nord Pool. The market is cleared each hour according to the competitive equilibrium model. The actual price and quantities for each hour are then communicated back to the participants.

The Nord Pool futures market is organized as a futures market having the spot market price as the underlying reference price. The contracts have a time resolution of one week with no physical delivery. Contracts with delivery up to three years can be traded. Contracts that mature after more than 5–8 weeks are stacked into blocks of 4 weeks. Contracts that mature after more than a year are stacked into seasonal contracts of 4–6 blocks. As the maturity of a block draws nearer, the block is dissolved and new ones created.

The main difference between futures (Nord Pool) and forwards (OTC) is the method of settlement. Futures are settled daily, marking to market. Forwards are settled during the delivery period of the contract. In our model we do not distinguish between the two types, and denote both types as forward contracts.

3. Portfolio modeling. Power scheduling is often modeled with one aggregated equivalent reservoir. Having the reservoir level as the state variable, stochastic dynamic programming (SDP) is employed to solve the problem, for a survey see [18]. This approach can handle stochastic prices, however a problem is how to de-aggregate the reservoir decisions. Approximate methods for multireservoir systems have been investigated, see for example [16] and [14]. Operations scheduling in deregulated markets with simplified contracting is discussed in [13]. The focus is on imperfect competition due to the low number of suppliers in New Zealand. The multireservoir hydro-thermal scheduling problem is also presented in [11]. It is demonstrated that if the number of stages is limited, a nested Benders decomposition algorithm can solve the problem without aggregation. Although not shown in that paper, stochastic spot prices can easily be incorporated. The multireservoir problem can also be solved by the Stochastic Dual Dynamic Programming (SDDP) algorithm [10], however with deterministic prices. Via a combination of sampling and decomposition that algorithm overcomes both the curse of dimensionality of traditional SDP and the number-of-stages limitation of nested Benders decomposition. Introducing stochastic prices in that algorithm does not pose a problem as long as price is not a part of the state space of the model. However, price must be a part of the state space due to the strong autocorrelation in spot market prices. This issue is discussed in [6]. There an algorithm is introduced that can handle stochastic prices, via a combination of SDDP and SDP.

An integrated production scheduling and contract management problem is formulated. There are T time periods, or *stages*, as illustrated in Figure 1. Periods are time intervals between stages, which are discrete

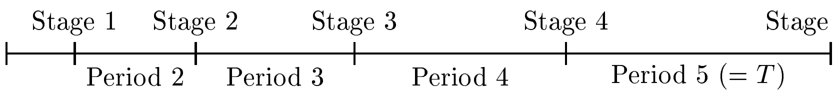

FIG. 1. *Example time scale*

points in time. The first period is deterministic. To simplify exposition, the problem is formulated for a producer with only one reservoir. This allows us to focus on the key feature of this model; the coordination of production and contracts under risk aversion.

The time periods of the model do not have to be of equal length. In the example in Section 6, the first two time periods are single weeks, the third period 11 weeks, the fourth period 26 weeks, and the fifth period 56 weeks. This structure could be changed to reflect the hydrological season.

The producer is operating an ongoing business with an indefinite future. We would like to avoid end effects, which are distortions in the model decisions due to the fact that the model has a finite horizon, whereas the real business problem has an indefinite horizon. For example, if in the model the value of the reservoir at the end of the model horizon is too low, say equal to zero, then the end effect would be that too much water is sold in the last stage. We propose two alternatives for this problem. One is choosing the date of stage T such that it makes sense to constrain the reservoir to be either empty or full at that date, i.e. in the spring before snowmelt, or in fall before winter. The other alternative requires estimating the end-of-horizon value of water in the reservoirs from a more aggregate model with a longer time span.

The stochastic variables are inflow, ν, spot price, π, and contract prices (φ for forwards and ω for options). They might be correlated; this is reflected in the scenarios. Scenarios are possible histories up to the end of the horizon. The event tree shown in Figure 2 shows how the uncertainty unfolds over time. A scenario in the event tree is a path from the root node

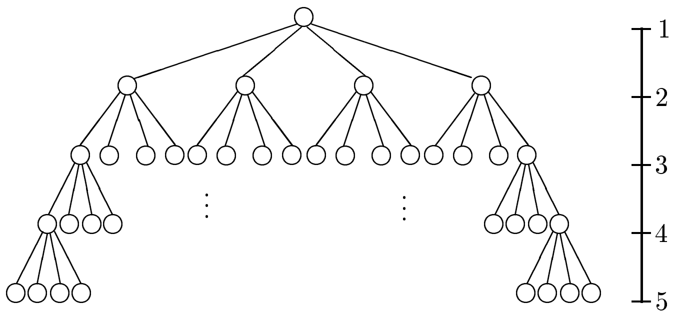

FIG. 2. *Example event tree and time scale when $T = 5$. The nodes represent decisions, while the arcs represent realizations of the uncertain variables.*

to a leave node. Each node n represents a *decision point*, or equivalently a *state*, corresponding to a realization of prices and inflows up to the stage of state n, denoted $t(n)$. The root state is $n = 1$, and scenarios are uniquely identified by states at the last stage, belonging to the set $\mathcal{S}$. The set of all states is denoted $\mathcal{N}$. The states have unconditional probabilities P_n, and every state except the root has a parent state $a(n)$. Let stage t decisions (for period t) be made *after* learning the realization of the stochastic variables for that stage.

The decision variables are reservoir discharge, u_n, spill, r_n, reservoir level x_n, and contracting decisions, which are discussed below. Each variable in the problem is indexed by the state to which it belongs. Power generation is generally a nonlinear function of the height of the water in the reservoir and the discharge, and could be non-convex. However, in our example we disregard head variation effects, and assume that generation is proportional to flow through the station, ρu_n, where ρ is the constant hydro-plant efficiency.

Let $V(x_n)$ be the value of the reservoir at the end of the horizon as a function of the reservoir level. This function must be specified to avoid end effects. If a long term scheduling model is available, V may be extracted from this model, e.g. in the form of incremental value of stored water in reservoirs. In most runs of our example the end-of-horizon reservoir level is fixed instead of using that function.

It is assumed that there is no direct variable cost of production, and that all power generated is valued at the spot price.

The hydro reservoir balance is

$$x_n - x_{a(n)} + u_n + r_n = \nu_n, \tag{3.1}$$

$$M_{INU,t(n)} \leq u_n \leq M_{AXU,t(n)}, \tag{3.2}$$

$$M_{IN\_X,t(n)} \leq x_n \leq M_{AX\_X,t(n)}, \tag{3.3}$$

for $n \in \mathcal{N}$ and with initial reservoir level given. Upper and lower limits on release and reservoir level are imposed using the bounds (3.2) and (3.3).

Three contract types are introduced into the model, namely forwards, options and load factor contracts. For the forward contract type there are few special difficulties. Of course, we must make sure that the prices of these contracts are statistically consistent with the spot price movements. The delivery profile of forwards has a constant power level during the whole delivery period.

The position in a forward contract in state n having delivery in period $k(\in \mathcal{K})$ is denoted by f_{kn}. Let negative f_{kn} represent a short position. Purchases and sales of forwards are represented by the nonnegative variables $f_{B,kn}$ and $f_{S,kn}$ defined for $k > t(n)$. The prices of these contracts are denoted φ_{kn}—for state n and delivery in period k. Assume that the prices are not influenced by the trading decisions, i.e. there is infinitely liquid and perfectly competitive markets. The contract level, or position, accumulated in state n is

$$f_{kn} = f_{k,a(n)} + f_{B,kn} - f_{S,kn}, \tag{3.4}$$

for $n \in \mathcal{N}$ and $k > t(n)$, with the initial forward position given.

Rebalancing decisions are made at each stage t, after the realizations of the random variables for period t are known. Transaction costs are proportional and utilize the coefficient T_F.

Option contracts are also included. We use the set $\mathcal{O}$ for calls and puts and $\mathcal{L}_{t(n)}$ for strike prices. Option prices and final payoffs are denoted by ω_{klon}. For positions and trading we use d and for transaction costs T_O. Otherwise the options are treated like forwards, and the rebalancing equation is not shown. Both option prices and forward prices are derived from spot prices, in a manner to be explaned in Section 4.

A basic feature of the model is risk aversion; we mainly support the hedging decisions of the producer. Modeling of risk is dependent on the views of the decision maker. Decision makers perceive risk as the potential for downside losses [12]. A way of accomodating this in a model is to have target levels for financial performance at different stages. The extent to which these targets are not met is called target shortfall, and one would progressively penalize target shortfalls in the objective, e.g. in the form of a piecewise linear cost function as shown in Figure 3. This way of penalizing operational risk has been successful in asset and liability models [19]. Let $m(\in \mathcal{M})$ be an index for the linear segments of the target shortfall variable (assume all targets have the same number of linear segments), and let $C_{m,t(n)}$ be the marginal shortfall cost in segment m. Let W be a weight parameter and denote s_{mn} the shortfall. The following inequality defines the shortfall variables:

$$q_{tn} + \sum_{m \in \mathcal{M}} s_{mn} \geq T_{ARGt}, \tag{3.5}$$

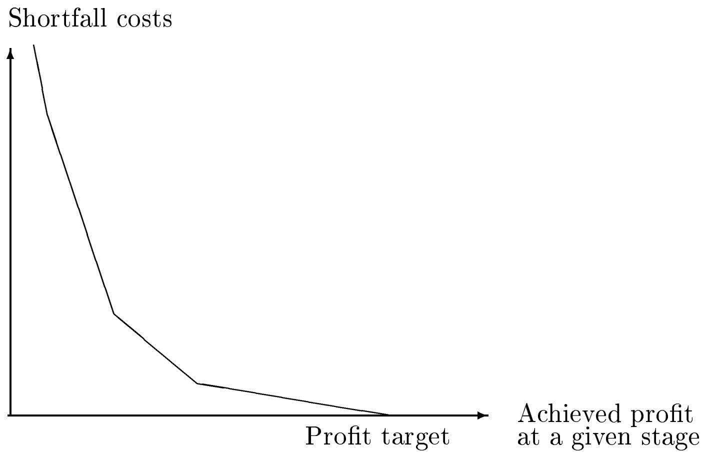

FIG. 3. *Shortfall cost function*

for all $n \in \{n : t(n) = t\}$ and all t for which there is a profit target T_{ARGt}, and where q_{tn} is the accumulated profit for period t in state n. It is given as

$$(3.6) \quad q_{tn} = \begin{cases} q_{t,a(n)} + (\varphi_{tn} - T_F)\, f_{S,tn} \\ -\, (\varphi_{tn} + T_F)\, f_{B,tn} & \text{if } t(n) < t \\ q_{t,a(n)} + \pi_n\,(\rho u_n + f_{t-1,n}) \\ + \sum_{l \in \mathcal{L}_t} \sum_{o \in \mathcal{O}} \omega_{tlon} d_{t-1,lon} \\ + \sum_{\substack{k \in \mathcal{K} \\ k > t}} \sum_{l \in \mathcal{L}_t} \sum_{o \in \mathcal{O}} \Big[(\omega_{klon} \\ -T_O) d_{S,klon} - (\omega_{klon} + T_O)\, d_{B,klon}\Big] & \text{if } t(n) = t, \end{cases}$$

for all $n \in \{n : t(n) \leq t\}$.

The objective function has six parts; net sale in the spot market, selling and buying forwards, selling and buying options, payoff from options, shortfall costs and value of the end reservoir.

$$\begin{aligned} \max \sum_{n \in \mathcal{N}} P_n (1+\gamma)^{-N_{t(n)}} \Big\{ & \pi_n \left(\rho u_n + f_{t(n)-1,n}\right) \\ + \sum_{\substack{k \in \mathcal{K} \\ k > t(n)}} & (1+\gamma)^{N_{t(n)} - N_k} \left[(\varphi_{kn} - T_F)\, f_{S,kn} - (\varphi_{kn} + T_F)\, f_{B,kn}\right] \\ + \sum_{\substack{k \in \mathcal{K} \\ k > t(n)}} & \sum_{l \in \mathcal{L}_{t(n)}} \sum_{o \in \mathcal{O}} \left[(\omega_{okin} - T_O)\, d_{S,klon} - (\omega_{klon} - T_O)\, d_{B,klon}\right] \\ + \sum_{l \in \mathcal{L}_{t(n)}} & \sum_{o \in \mathcal{O}} \omega_{t(n)lon} d_{t(n)-1,lon} \end{aligned}$$

$$
\begin{aligned}
&-W \sum_{m \in \mathcal{M}} C_{m,q(n)} s_{mn} \Bigg\} \\
&+\sum_{s \in \mathcal{S}} P_s (1+\gamma)^{-N_T} V(x_s),
\end{aligned}
\tag{3.7}
$$

where $\gamma \geq 0$ is a discount interest rate. The discount factor is adjusted for time periods having unequal length; N_t is the number of years from now until stage t.

In the first term of the objective function, the $f_{t(n)-1,n}$ variable represents net energy supply from forwards. It is the forward position before rebalancing. If the producer has any fixed commitments for power delivery it should be added there.

Company policy may restrict investment in contract categories. Limits on short sale, liquidity considerations and risk policies can often be expressed in the form of linear constraints. Illiquidity can also be incorporated as higher transaction costs.

Many of the contracts are financial in nature. These entitle or obligate the holder (the power purchaser) to receive or pay the difference between the spot price and the strike price, which is the agreed contract price. We model forwards as if all of them were contracts for physical delivery. Many forwards are settled financially, so in the first term of the objective function the variables for selling and buying are not equal to the physical exchange of power on the power pool spot market. For example, if a producer has hedged against a price decrease by short selling (financial) forwards or futures, the producer may end up having to physically sell power cheaply, but is compensated through the hedge contracts. Thus in terms of risk and expected return, the producer may have ended up financially buying power in a favorable price situation.

Our approach, where risk is penalized in the objective function through shortfall costs, yields a piecewise linear concave objective function in profit. The objective function is thus interpreted as a utility function that reflects risk aversion. See Figure 4. With this approach risk is incorporated in the objective, and the discount interest rate should not be adjusted for risk [1].

A special type of OTC contract is the so-called load factor contract. With these contracts the holder must continuously decide how much of the contract's energy shall be released. For example, the 5000 hour contract mentioned above can be employed at the maximum power level (which is given in the contract) for no more than 5000 hours of the total 8760 hours

[1] It can be argued that this objective function is too simple to be interpreted as an intertemporal utility function, and thus discounting should in some way reflect risk. However, our approach must be considered in the light of our criterion for choosing an objective function, namely, what is computationally feasible and what is acceptable for the decision maker.

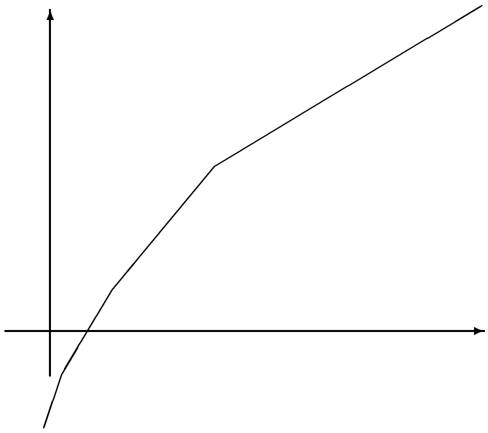

FIG. 4. *Piecewise linear concave utility function*

in the contract delivery period. The holder can choose not to release the total contract volume, but should always do that because the electricity price is positive.

Let $r_{EL\_LFCn}(\geq 0)$ be the energy released from the contract at state n, $l_{FCPOS,n}(\geq 0)$ the volume of the contract, $M_{AX\_LFC\_REL,t}$ the maximum release in period t, $\mathcal{U}$ the set of summer periods in the contract's delivery period and $\mathcal{W}$, the set of winter periods. The node at which the trading period ends and delivery period begins is denoted $D(n)$. This is for a given contract, i.e. with a specified delivery period and load factor (for example, 5000/8760). The release in period t becomes part of the power balance and is thus valued in the objective function at the current spot price. The following constraints, valid for the delivery period of the contract, ensure that the release is according to the contract terms:

$$l_{FCPOS,n} - l_{FCPOS,a(n)} + r_{EL\_LFCn} = 0 \tag{3.8}$$

$$r_{EL\_LFCn} \leq M_{AX\_LFC\_REL,t(n)} \tag{3.9}$$

$$\sum_{\mathcal{U}} r_{EL\_LFCn} = l_{FCPOS,D(n)} F_{RACS} \tag{3.10}$$

$$\sum_{\mathcal{W}} r_{EL\_LFCn} = l_{FCPOS,D(n)} F_{RACW} \tag{3.11}$$

where F_{RACS} and F_{RACW} is the fraction of the volume to be released in summer and winter respectively, and $F_{RACS} + F_{RACW} = 1$. The term

$(\pi_n - P_{LFC})r_{EL\_LFC,n}$ must be added to the objective function, where P_{LFC} is the load factor contract price. In the trading period of the contract the ordinary rebalancing apply and is similar to Equation (3.4).

With short sale of load factor contracts, we must make assumptions about how the holders of contracts that we have sold will behave regarding release from the contract over time. One such assumption could be to assume that these holders will in aggregate behave as if they were risk neutral.

4. Scenario Generation. The generation of scenarios involves considerable effort in large-scale stochastic programming models. In Norway there is more than 60 years of observed data on inflow, and a spot market has been in operation for more than 25 years. However, there was restricted access to this market before the deregulation effective January 1, 1991. Forecasting spot prices and inflow is not a new activity, but forecasting the prices of forwards and futures is more recent. Observed market prices for futures on electricity are available only from the last few years. The Scandinavian futures market has had periods of very limited liquidity, which not only makes the historical data less appropriate for forecasting purposes, but also creates a need in portfolio management to limit the sizes of purchases and sales of these contract types. However, the liquidity problems are becoming so small that they are not worth modeling.

4.1. Price Forecasting. The Multiarea Power Scheduling (MPS) model is a market equilibrium model frequently used for price forecasting in Scandinavia. This model was developed by SINTEF Energy Research and is described in [3, 2]. In the MPS model, process submodels describe production, transmission and consumption activities within the Nordic and adjacent areas. The various demand/supply regions are connected through the electrical transmission network. A solution of the model results in a set of equilibrium prices and production quantities, for each week over the time horizon considered (usually 3 years) and for each historical inflow year. The demand side of the model consists of price dependent and price independent load for each region. Important input for the model is demand, thermal generation costs, and initial reservoir levels. The model is short term in the sense that there are no mechanisms for endogenously increasing production capacity. The MPS recognizes that hydro scheduling decisions are made under the uncertainty of reservoir inflows; to determine the opportunity hydro generation costs, and production in each region, stochastic dynamic programming is employed on the scheduling problem where production in the region is aggregated into an equivalent reservoir/power station pair.

The MPS model generates independent scenarios for price and inflow. However, this structure is not appropriate for multistage stochastic programming. What is needed is a *scenario tree* where information is revealed in all stages of the model.

4.2. Risk Adjustment. The MPS model strives to find equilibrium prices according to an expected social optimum criterion. Such a solution would occur in the electricity market if all market participants were risk neutral and price taking. The interest rate used for discounting cash flows in that model is assumed to be the risk free rate of interest, and so the probabilities coming from this model can be interpreted as being "risk neutral". This means that we can easily adopt so-called risk neutral valuation principles to pricing of contracts and portfolios. Consistent with this, we require the expected average spot prices from MPS for future periods corresponding to delivery periods of traded contracts in the term markets to equal the currently observed prices of these contracts. The discount interest rate we use for all cash flows and contract payoffs is the risk free one. For example, options would be priced according to their expected discounted payoff. In practice it is necessary to adjust the MPS scenarios (up or down) so the "term structure" that can be derived from the MPS scenarios equals the observed term structure of futures prices.

Contrary to what is common in Stochastic Programming, we optimize over a risk neutral probability measure. As modern financial theory dictates, the appropriate discount interest to use is the risk free rate. But what does it mean to optimize using a risk averse objective function over a risk neutral event tree? The alternative is to use the empirical probability measure and an appropriate discount factor[2]. In order to study the effects of the choice of probability measure we notice that it enters only in the objective function of the model. The objective function has two major parts: the net present value of the portfolio and the risk costs. The net present value part does not constitute a problem, because that is found in a manner consistent with modern valuation theory. The question of how the risk cost is affected remains. Assuming that electricity market risk is positively correlated with overall market risk, going from an empirical measure to a risk neutral measure means moving probability mass from "rich" states for an average hydropower producer over to "poor" states. This is because under the risk neutral measure, all investment opportunities have expected return equal to the risk free rate, while under the empirical measure it is reasonable to assume that risky investment opportunities have relatively more probability mass in "rich" states. Since "poor" states generally have high shortfall costs, this means that for an average producer, the recommended decisions would be more risk averse than under the alternative with a constant discount rate and the empirical probability measure.

In the model, forward prices equal the conditional expected spot price for the delivery period. To maintain consistency with the market, the decision maker should set the spot price scenarios so that the expected spot

[2]The most common practice is to use a constant discount factor. Another choice of discounting would be to use the risk neutral probabilities as basis for finding stochastic discount factors, but then the two approaches would be equivalent.

price for a period equals the current market price of a forward with delivery in that period. Posing this constraint on scenario generation means that the model supports solely the hedging aspect of trading in contracts. If the decision maker expects the average spot price to be different from the forward price in any period, there is a speculative motivation for entering into a position in that contract. This aspect of contracting is important to some producers, but the procedure indicated above does not give a model that supports that aspect. The advantages of doing this lies in the importance for risk control and reporting to separate between hedging and speculation. An alternative approach would be to allow for a gap between the forward price and the expected future spot price that gradually is diminished as the time to maturity approaches. In a practical application, such a feature would be valued, because most producers also make speculative trades.

Contract prices are set equal to their conditional expected discounted payoff. We discount all cash flows using the risk free rate. The forward contract is priced as:

$$\varphi_{kn} = \frac{\sum_{m \in \mathcal{F}_{kn}} P_m \pi_m}{\sum_{m \in \mathcal{F}_{kn}} P_m}$$

where $\mathcal{F}_{kn}$ is the set of all decendant states of state n belonging to stage k, i.e. all states at stage k having n as ancestor. Option prices, for example in the case of a call, are calculated as:

$$\omega_{kl,call,n} = \frac{\sum_{m \in \mathcal{F}_{kn}} P_m \max(0, \pi_m - X_{kl})}{(1+\gamma)^{N_k - N_{t(n)}} \sum_{m \in \mathcal{F}_{kn}} P_m}.$$

4.3. Generating Scenario Trees. The applied scenario generation method is a combination of simulation and construction, see [9]. The decision maker specifies the market expectations using statistical properties that are considered relevant for the problem, and constructs a tree with these statistical properties. Some statistical properties are state dependent, while others are independent. As an example of state dependency, consider autocorrelation of spot market prices. If prices have been high in the previous period, then it is likely that prices in the following period are high also. We model this effect by letting the price in period $t+1$ be a function of the outcome in period t.

5. Validation. To gain acceptance, a model must be tested and validated. The tests should verify that the model performs according to its specifications. Validation means proving that the model performs better than its alternatives by some accepted criteria.

Of particular importance for stochastic programming models is testing for stability. With a small change in the input data, the resulting optimal solution should be very close to the original solution, either in terms of the objective function, or in terms of the decisions, or both. The objective funtion could be relatively flat, in which case one can not expect the optimal decisions to be stable. The converse may also happen, namely a variable objective function level with relatively stable optimal decisions. In both of these cases the overall model should be declared stable.

For validation, what is important is the relative performance of our model compared to alternative decision support tools. In our context the most advanced alternative is to alternate between a (risk neutral) hydro scheduling model and a myopic portfolio model. In such a scheme, one would first schedule production without regard to risk or contracts. This schedule would serve as input in a static contract portfolio tool. Using a mean-risk criterion, this tool seeks trading decisions of modeled contracts without regard to future rebalancing. There is no value of waiting in this model, and decisions will be made as if here and now is the only chance for mitigating risk through contracting. Thus, there are three sources of suboptimality in this procedure. One stems from the fact that some of the decisions are made using a different objective function than the correct one. The second source relates to the weak coordination between hydro scheduling decisions and trading, and the third to the lack of dynamics for contract optimization.

To quantitatively assess the relative performance of the two models one may utilize a simulation model that incorporates both types. Employing rolling horizon simulations for many scenarios, the two models would be rerun at regular intervals for a long time period. Further testing and validation issues are discussed in terms of the numerical example.

6. Numerical Example. We consider a producer having 7 hydro plants and 11 reservoirs. Average production is 2500 GWh, storage capacity is 1490 GWh, and generation capacity is 595 MW. The reservoirs are presently on average 65% full. The present portfolio includes one load factor contract. The producer has sold a large amount of fixed contracts, so that in expectations, he is buying 192 MW in the spot market. The decision maker wants to find the optimal release from the reservoirs, and the optimal buying and selling of contracts which have delivery in some critical future periods.

The reservoirs are situated along two river systems, cf. Figure 5. There is uncertainty in inflow into the two rivers and spot market prices. We employ a five period (five stage) model with 256 scenarios. The first two periods have a length of one week, the third 11 weeks, then 26 and 56 weeks.

The basis for generating the scenarios is

- user supplied statistical moments for the first period marginal distributions of all random variables,
- correlation between the variables,
- definition of the state dependent statistical properties and
- bounds on outcomes and probabilities.

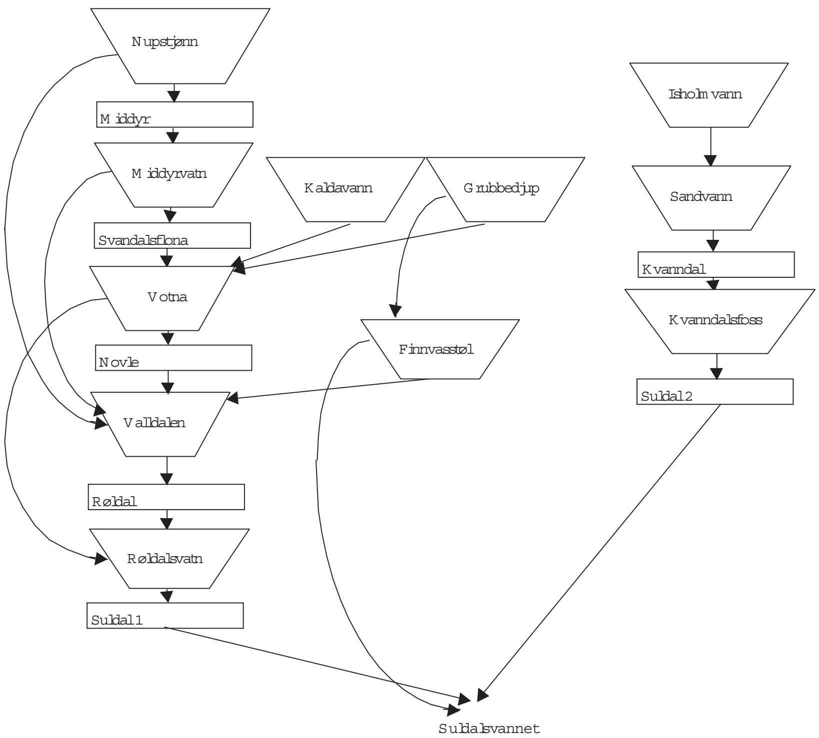

FIG. 5. *The Røldal and Suldal river power system. Trapezoids represent reservoirs, and rectangles represent power stations. Arched lines represent spillways, and are only shown when the spillway is different from the station watercourse (straight lines).*

Price forecasting and analysis of historical inflow data was done using the MPS model. We assume that the first three moments and correlations are the relevant statistical properties. The specifications are given in Tables 1 and 2.

We have modeled state dependent expected values and standard deviations for all uncertain variables. The numbers in Table 1 are the *unconditional* specifications for these properties. The other statistical properties are assumed state independent, so they are the same in all states of the world at a certain point in time.

The state dependent mean in period $t > 2$ is

$$E(x_{it}) = E_{BAS}(x_{it}) + AC_{it}\frac{SD_{BAS}(x_{it})}{SD(x_{i,t-1})}(x_{i,t-1} - E(x_{i,t-1})), \tag{6.1}$$

where $E(x_{it})$ is the expected outcome of random variable i in period t, $E_{BAS}(x_{it})$ is the average (basis) expected value given in Table 1 for random variable i in period t, $SD_{BAS}(x_{it})$ is the corresponding average standard

TABLE 1
Specifications of market expectations. Period 1 is deterministic.

Stoch. param.	Distr. property	Period 1	2	3	4	5
Spot market price	Exp. NOK/MWh	187.6	187.4	187.2	167.0	175.9
	std. dev.		1.50	7.00	35.0	35.0
	skewness		-1.26	-1.43	-0.98	0.13
Inflow river 1	exp. value	3.54	3.34	27.3	812.3	1137
	std. dev.		3.10	23.5	170	231
	skewness		2.55	3.00	0.10	-0.88
Inflow river 2	exp. value	3.28	3.27	21.7	827.9	1128
	std. dev.		2.70	20.0	180.0	232
	skewness		1.90	3.90	0.20	-0.9

TABLE 2
Specification of correlations.

Correlation	Period 2	3	4	5
Price–Inflow river 1	-0.73	-0.88	-0.90	-0.57
Price–Inflow river 2	-0.74	-0.80	-0.90	-0.58
Inflow river 1–Inflow river 2	0.84	0.88	0.98	0.99

deviation, x_{it} is the outcome of random variable i in period t, and $AC_{it} \in [-1, 1]$ is an autocorrelation factor (a large AC_{it} leads to a high degree of autocorrelation).

For standard deviation we assume that the state dependency is

$$SD(x_{it}) = SD_{BAS}(x_{it})(1 - AC_{it}^2). \tag{6.2}$$

In Table 3 the autocorrelation factors are listed.

We bound the outcomes at the minimum and maximum observed in the underlying data. We also specify bounds on probabilities, ensuring that scenario probabilities are reasonably uniform.

The first four stages of the tree are shown in Figure 6. Careful examination of the numbers in the figure will reveal that the specifications in the tables above are not met 100%. This is probably due to overspecification; too many statistical properties are to be satisfied relative to the size of the tree.

Generating several scenario trees and subsequently solving the stochastic programming problem gives reasonable stability in terms of objective function values and aggregated first stage decisions. Some contract decisions are somewhat unstable however, ranging from 0 to 6 times expected generation in the delivery period of the respective contracts. The correlation of these decisions and statistical properties that were not specified

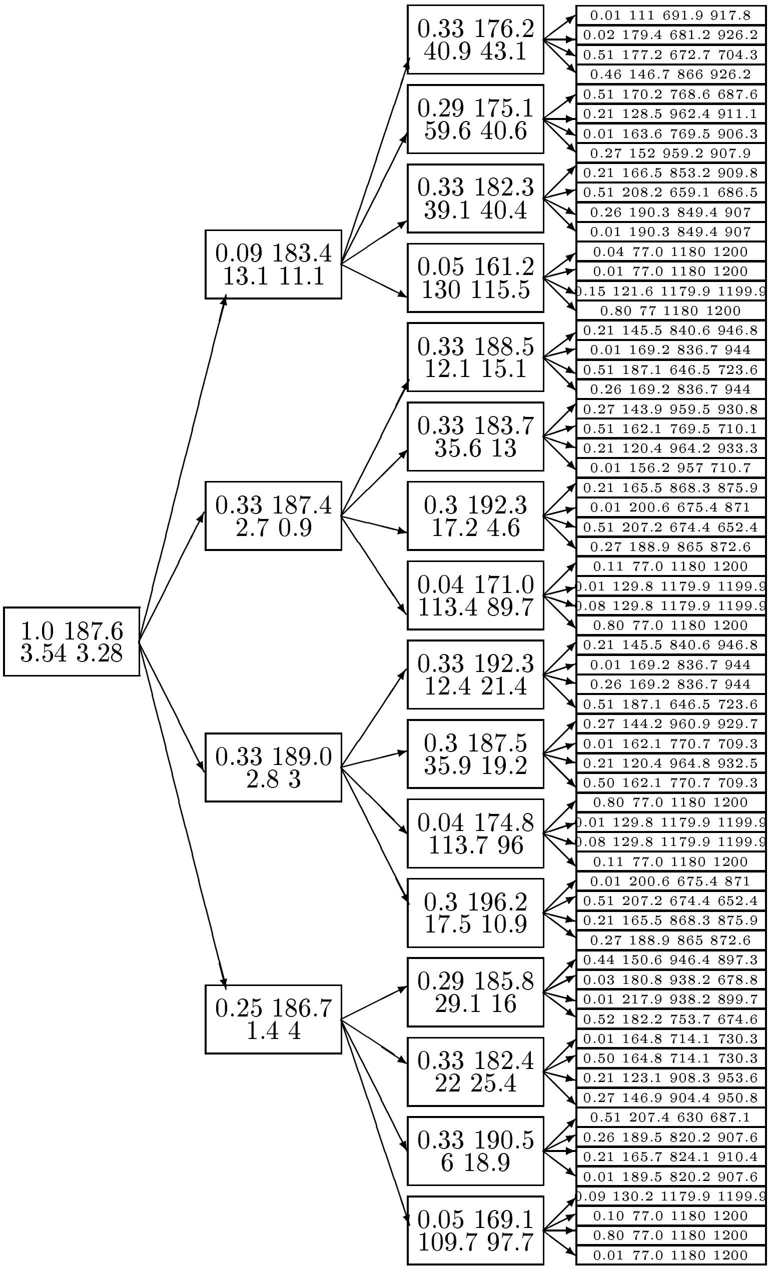

FIG. 6. *The generated event tree. The last period is not shown. The numbers in the boxes represent conditional probability, average spot market price for the period in NOK/MWh, inflow to reservoir 1 and 2. The first box on the left is deterministic and represent the outcome in the period before the first stage. Thus the numbers in this box were not generated in the procedure.*

TABLE 3
Autocorrelation factors defining state dependencies.

	Period		
Uncertain variable	3	4	5
Spot market price	0.60	0.40	0.20
Inflow river 1	0.31	0.13	0.025
Inflow river 2	0.41	0.20	0.02

in the scenario generation, such as kurtosis of all random variables, higher order cross terms etc. were close to zero. Thus specifying these statistical properties would not lead to increased decision stability. Furthermore, these particular contract decisions only have a very small impact on expected porfolio value as well as on shortfall costs.

Profit target shortfall is measured and penalized in stages 3 to 5. There are four forward contracts, with delivery in periods 2 to 5, respectively. There is a 0.18 NOK/MWh transaction cost on both buying and selling of forwards and options. There are 8 put and 8 call contracts, maturing at stages 2 to 5, i.e. two puts and two calls for each delivery stage. Due to liquidity considerations, prices are raised (lowered) by 3.5% for buying (writing) options, and for forwards the corresponding number is 0.5%[3].

The objective (Equation (3.7)) is maximized for different weights W on the shortfall costs. To mitigate the effect of the possibly incorrect specification of the value of the water in the reservoir at the model horizon, $V(x_s)$, we set target levels for the end-of-horizon reservoir levels, one for each scenario. We found this target by solving first with no weight on the shortfall costs, i.e. a risk-neutral run, with the value of the reservoir set at spot market prices. In all subsequent runs in this paper, these target reservoir levels are used. Figure 7 displays points on the efficient frontier.

The risk neutral point at the high right end of the graph, has a risk that is 7.7 times higher than at the minimum risk point at the low left end of the graph. The expected profit is only 1.7% higher. We conclude that for a hydropower producer, employing a dynamic stochastic model with risk aversion and forward and option contracts, it is possible to reduce risk significantly compared to a risk neutral approach without contracts, and only losing marginally in terms of expected profit.

6.1. The performance of static portfolio approaches. The current industry practice is to schedule production without contracts first, and decision support for contract trading is based on static portfolio models. The two approaches should ideally have been compared using rolling horizon simulations as in [4]. This reflects that in both the dynamic and

[3]These coefficients are diminishing gradually. Unfortunately, the results presented are for slightly higher liquidity premia.

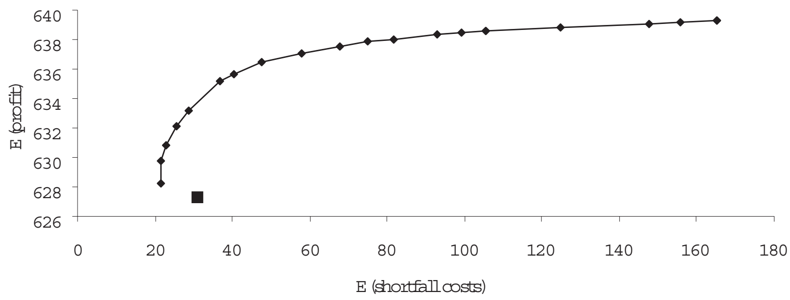

FIG. 7. *The efficient frontier displays the tradeoff between expected portfolio profit and risk, and is obtained by solving the model for different weights on the shortfall costs in the objective function.*

static approaches, the decision maker uses only the first stage decision, and then reruns the model based on new information. A simpler approach to comparison is adequate for this example.

The performance of the static approach in terms of expected profit and risk at the end of the horizon can be found approximately by first finding decisions in the following way:

- The model is run without contracts, and a risk neutral production strategy is obtained. The model is then rerun with the production strategy found above kept fixed, with buying and selling allowed only for the first stage.
- For each stage following, the model is rerun with buying and selling allowed only for the current stage. This is repeated until stage 4, where the contract with delivery in the last period is last traded.

This means that at any stage, the model only sees a now-or-never opportunity for trading.

The resulting point in the mean-risk diagram is shown as the square off the frontier in Figure 7. The reduction in total objective function value is 2.4%, and one can obtain a 1.1% increase in expected profit with the same level of risk when employing a dynamic approach instead of a static one. We conclude that a dynamic stochastic model can add value to portfolio management.

The first stage decision for the dynamic approach regarding forwards, was to buy $(0, 0, 3392, 0)$ GWh for the four delivery periods. For the static approach, the corresponding purchase was $(0, 1374, 2532, 6381)$ GWh. The recommended option trade was 31% higher. This larger trading volume is due to the fact that in the static approach, the model does not see the value of waiting for more information, so that unnecessary transaction costs can be avoided. All risk that can be dealt with through the forwards, must be mitigated in the first (current) stage.

7. Implementation issues. The development of this model has been bifurcated. After the initial publication of a general framework [5], there has been substantial industry interest, and in addition to the work reported here, a commercial prototype of the model has been implemented for Norsk Hydro by SINTEF Energy Research. The specification of this prototype was based on a development effort by the Norwegian University of Science and Technology, SINTEF and Norsk Hydro [7]. It is currently in use at Norsk Hydro Energy for decision support.

Among algorithms that can be used to solve the model, Benders' decomposition [1], also referred to as the L-shaped decomposition method [17], stochastic dual dynamic programming (SDDP), or a combination of different decomposition schemes, are best suited. An SDDP variant [6] is used in the commercial prototype of the model. The idea of the SDDP algorithm is to store the future cost function of dynamic programming in the form of nested Benders cuts instead of in a table, which is usual in SDP. This overcomes the curse of dimensionality. The state variables are the hydro reservoir levels and the trend in stochastic inflow and spot market price. At any stage, all state variables except price are related through linear functions. Thus the future cost function of the previous stage is convex in these state variables. However, the price state variable is related to reservoir levels and inflow through a product term making the overall future cost function for this stage nonconvex. This issue is resolved by using price as a "super" state, building separate future cost functions for each price state at each stage.

Test cases have been run at SINTEF for a 104-stage problem (two years, weekly resolution) having 11 reservoirs and 21 different contracts (forward type only, having different delivery periods). The CPU time to solve these problems is 3-4 hours.

Large-scale linear programs can also be solved by commercial optimization packages such as CPLEX and IBM's OSL. The recent advances in interior point methods and the simplex method makes this approach an alternative to decomposition. We have implemented the example as a large scale deterministic equivalent LP in AMPL, using CPLEX 6.0 as solver. The numerical example takes about 15 seconds to solve on a 200 MHz workstation.

8. Further Development. As in any model, many aspects of the real system under study have been omitted to focus on particularly interesting aspects. We wanted to highlight the coordination of physical generation resources and financial instruments such as forwards, i.e. portfolio management. Several issues should still be examined before the model can be fully specified and then implemented and solved. For example, some producers may control large parts of the total power supply, or there can also exist dominating buyers. We have assumed that the scheduling decisions made by the producer under study does not affect the uncertainty in prices or other random variables. Dominating producers, as in the UK and Scandinavia, may be able to distort spot prices and thereby inhibit the efficient operation of futures markets. How this affects electricity portfolio management needs to be determined.

Most end users have contracts where they can consume as much as they desire at the contract price, changeable in two weeks' notice. Total demand facing a vertically integrated utility under such contracts is uncertain but correlated with the spot price. For utilities with significant volumes of such contracts, such end user dynamics should be incorporated into portfolio management. In our framework such demand could be treated as a special contract category with random volumes.

Our model is basically energy-oriented. In power systems based largely on thermal production, one needs to be more power (capacity) oriented, also in portfolio management. One needs to make sure that all power trade is within physical limits. Thus it seems necessary to have a finer time granulation, possibly using time segments such as peak, medium and low load.

Transmission network aspects have been ignored in this model. In many systems this is unrealistic, there could be significant spatial risk. In such cases transmission congestion contracts are a natural part of the hedging opportunities considered in portfolio management.

In Norway, there are tax issues causing distortions in the production decisions. These tax rules should ideally be incorporated in a portfolio model. Also, the issue of maintenance and forced outages, and existence of pumped storage units, have been ignored. The scenario generation also needs further development.

9. Conclusion. This paper presents a model for portfolio management in a deregulated hydropower based electricity market. A general framework has been formulated, and major issues discussed. Many aspects remains to be developed. For example, for the Scandinavian and UK markets, some producers are so large that the assumption of perfect competition in production is not realistic.

The presence of markets for electricity makes it necessary for power producers to coordinate physical generation resources with the trading and financial settlements of "paper" resources such as forward contracts or other types of derivatives that can replace physical deliveries and mitigate the risk associated with fluctuating prices on electricity. The industry practice is to use dynamic stochastic models for production scheduling, and static models for contracts, running these sequentially. A stochastic programming implementation of the integrated dynamic model run on an example portfolio shows that risk can be reduced by about 32% (for the same level of expected profit) compared to the industry practice. At the same level of risk, the expected profit can be increased by 1.1%.

REFERENCES

[1] J. F. Benders, *Partitioning procedures for solving mixed-variables programming problems*, Numerische Mathematik, 4 (1962), pp. 238–252.

[2] O. J. Botnen, A. Johannesen, A. Haugstad, S. Kroken, and O. Frøystein, *Modelling of hydropower scheduling in a national/international context*, in Hydropower '92, E. Broch and D. Lysne, eds., Lillehammer, Norway, June 1992, Balkema, Rotterdam, pp. 575–584.

[3] O. Egeland, J. Hegge, E. Kylling, and J. Nes, *The extended power pool model—Operation planning of multi-river and multi-reservoir hydrodominated power production system—a hierarchial approach*, Report 32.14, CIGRE, 1982.

[4] S.-E. Fleten, K. Høyland, and S. W. Wallace, *The performance of stochastic dynamic and fixed mix portfolio models.* Working paper, Department of Industrial Economics and Technology Management, Norwegian University of Science and Technology. Available: http://www.iot.ntnu.no/iok_html/papers., 1998.

[5] S.-E. Fleten, S. W. Wallace, and W. T. Ziemba, *Portfolio management in a deregulated hydropower-based electricity market*, in Hydropower '97, E. Broch, D. Lysne, N. Flatabø, and E. Helland-Hansen, eds., Trondheim, Norway, July 1997, Balkema, Rotterdam, pp. 197–204.

[6] A. Gjelsvik and S. W. Wallace, *Methods for stochastic medium-term scheduling in hydro-dominated power systems*, Report EFI TR A4438, Norwegian Electric Power Research Institute, Trondheim, 1996.

[7] A. Grundt, B. R. Dahl, S.-E. Fleten, T. Jenssen, B. Mo, and H. Sætness, *Integrert risikostyring (integrated risk management)*, 1998. Energiforsyningens Fellesorganisasjon-Pub. nr. 255, Oslo.

[8] A. Haurie, Y. Smeers, and G. Zaccour, *Toward a contract portfolio management model for a gas producing firm*, INFOR, 30 (1992), pp. 257–273.

[9] K. Høyland and S. W. Wallace, *Generating scenario trees for multi stage decision problems*, Management Science, (1999). To appear.

[10] M. V. F. Pereira, *Optimal stochastic operations scheduling of large hydroelectric systems*, International Journal of Electrical Power & Energy Systems, 11 (1989), pp. 161–169.

[11] M. V. F. Pereira and L. M. V. G. Pinto, *Stochastic optimization of a multireservoir hydroelectric system—a decomposition approach*, Water Resources Research, 21 (1985), pp. 779–792.

[12] W. J. Petty, D. F. Scott, and M. M. Bird, *The capital expenditure decision-making process of large corporations*, The Engineering Economist, 20 (1975), pp. 159–172.

[13] T. J. Scott and E. G. Read, *Modelling hydro reservoir operation in a deregulated electricity sector*, International Transactions in Operations Research, 3 (1996), pp. 209–221.

[14] V. R. Sherkat, R. Campo, K. Moslehi, and E. O. Lo, *Stochastic long-term hydrothermal optimization for a multireservoir system*, IEEE Trans. Power Apparatus and Systems, PAS-104 (1985), pp. 2040–2049.

[15] S. Stage and Y. Larsson, *Incremental cost of water power*, AIEE Transactions (Power Apparatus and Systems), August (1961), pp. 361–365.

[16] A. Turgeon, *Optimal operation of multireservoir power systems with stochastic inflows*, Water Resources Research, 16 (1980), pp. 275–283.

[17] R. van Slyke and R. J.-B. Wets, *L-shaped linear programs with applications to optimal control and stochastic programming*, SIAM Journal of Applied Mathematics, 17 (1969), pp. 638–663.

[18] S. Yakowitz, *Dynamic programming applications in water resources*, Water Resources Research, 18 (1982), pp. 673–696.

[19] W. T. Ziemba and J. M. Mulvey, eds., *Worldwide Asset and Liability Modeling*,

Cambridge University Press, Cambridge, U. K., 1998.