The Application of Operations Research Techniques to Financial Markets

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ABSTRACT

This paper reviews the application of OR to financial markets. After considering reasons for the attractiveness of general finance problems to OR researchers, the main types of financial market problem amenable to OR are identified, and some of the many problems solved using OR are documented. While mathematical programming is the most widely applied technique, Monte Carlo and other simulation methods are increasingly widely used. OR now plays an important role in the operation of financial markets and this importance is likely to increase, creating the opportunity for OR (and operations researchers) to play an even greater role.

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Operations Research, OR, has been applied to problems in finance for at least the last half century. The INFORMS data base of academic papers in OR journals since 1982 classifies nearly 3% of the entries as being concerned with finance. For the journal *Management Science* over the same period this proportion is over 10%. There is an even larger number of papers on the application of OR techniques to finance in the finance, mathematics, engineering and other literatures, so that, in total, there are several thousand papers which apply OR techniques to finance in academic journals. Equally, OR has played a part in the adoption by the financial markets of the new finance theories. For example, in the 1960s and 1970s the Management Science group at Wells Fargo Bank in San Francisco pioneered the used of new finance theories, and introduced the first index tracking fund in July 1971 (Bernstein, 1992). As part of the increased use of mathematical models in finance (Merton, 1995), investment banks have recruited staff skilled in quantitative techniques, including OR, to devise pricing equations and analyse market data - the so called “quants” or “rocket scientists”.

This paper considers the application of OR techniques to financial markets. This covers decisions concerning trading by decision makers in financial markets (e.g. the debt, equity and foreign exchange markets and the corresponding derivatives markets), and represents a more recent and still growing area for the application of OR techniques to finance. This paper does not consider the more traditional applications of OR to the management of the firm’s finances: working capital management (which can be subdivided into the management of cash, receivables and liabilities), capital investment (including the appraisal and implementation of sets of large interdependent investments), multinational taxation, and financial planning models (such as those developed for banks); which have been reviewed by Ashford, Berry and Dyson (1988). Models for forecasting movements in financial markets and bankruptcy prediction are not considered, as they are deemed to be outside the scope of this paper.

After considering some of the reasons for the attractiveness of finance problems for the application of OR techniques, this paper identifies the main types of problem that are amenable to OR analysis, and documents some of the many problems in financial markets which have been addressed using OR techniques.

1. Attractiveness of Finance Problems

An important distinguishing feature of problems in financial markets is that they are generally separable and well defined. The objective is usually to maximise profit or minimise risk, and the relevant variables are amenable to quantification, almost always in monetary terms. In contrast to some other OR applications, the investigator has few worries about ensuring that they have identified the correct question (i.e. there is no need to consider whether the problem is to reschedule the company's vehicle fleet to meet customer needs, rather than the broader question of whether it needs to operate a fleet of vehicles at all). In finance problems, the relationships between the variables are usually well defined, so that, for example, the way in which an increase in the proportion of a portfolio invested in a particular asset affects the mean and variance of the portfolio is clear. Thus the resulting OR model is a good representation of reality, particularly as the role of non-quantitative factors is often small. Finance problems also have the advantage that any solution produced by the analysis can probably be implemented, while in other areas there may be unspecified restrictions concerned with human behaviour and preferences that prevent the implementation of some solutions. Furthermore, finance practitioners are accustomed to the quantitative analysis of problems.

The investigator is likely to find that much of the requisite historical data has already been collected and is available from company records or recorded market transactions, and that large amounts of real time data are available on prices (traded and quoted) in financial markets which can readily be used in OR models. In addition, non-quantitative factors are generally absent from formulations of finance problems.

The availability of real time data means that solutions can often be implemented very quickly (e.g. a few seconds) and, as trading in financial markets often involves very large sums of money, even a very small improvement in the quality of the solution (under 0.5%) is beneficial. Furthermore, such problems tend to recur, possibly many times per day, spreading the costs of developing an OR solution over a large number of transactions. This scale and repetition makes the development of an OR model more attractive than for small or one-off decisions.

Thus, because finance applications (especially applications to financial markets) are largely numerical problems with well defined boundaries and objectives, clear relationships between the variables, large benefits from very small improvements in the quality of decision making and excellent data, they are well suited to OR analysis. This paper analyses the application of OR techniques to financial markets in more detail by considering some of the major types of problem in financial markets, and the OR techniques that have been used to analyse them.

2. Portfolio Theory

A seminal application of OR techniques to finance was by Harry Markowitz (1952, 1987) when he specified portfolio theory as a quadratic programming problem (for a survey of this theory, see Board, Sutcliffe and Ziemba, 1999). Participants in financial markets usually wish to construct diversified portfolios because this has the substantial advantage of reducing risk, while leaving expected returns unchanged. The objective function for the portfolio problem is generally specified as minimising risk for a given level of expected return, or maximising expected return for a given level of risk. While returns produce a linear objective function, risk is modelled using the variance, leading to an objective function with quadratic variance and covariance terms. The Markowitz model also includes non-negativity constraints on the decision variables to rule out short selling of the asset concerned[1]. As well as specifying the portfolio problem within a mean-variance framework, Markowitz also developed solution algorithms for more general quadratic programming problems. This provides an example of the interaction between OR techniques and finance, with the former sometimes being tailored to meet the needs of the latter. Subsequently the idea of using the variance to model risk has been extensively used in finance, and hence in applications of OR to finance.

Although the most obvious application of portfolio theory is to the choice of equity portfolios, and empirical papers (e.g. Board and Sutcliffe, 1994; and Perold, 1984) have used quadratic programming to compute efficient equity portfolios, the technique can be applied to a much wider range of problems. Konno and Kobayashi (1997) proposed using quadratic programming to form portfolios of both equities and bonds. Other authors have been concerned with managing bond portfolios to maximize their expected value, and have used stochastic linear programming to allow for interest rate risk (e.g. Bradley and Crane, 1972). Golub et al. (1995), Zenios (1991, 1993b) and Zenios et al (1998) employed stochastic programming to select a portfolio of fixed interest securities (Mortgage Backed Securities, MBS[2]) that maximised the expected utility of terminal wealth, after using Monte Carlo simulation to generate the various scenarios, while Ben-Dov, Hayre and Pica (1992, used stochastic programming to form portfolios of MBS and other assets for clients that were expected to outperform some pre-specified target return.

Pension funds hold portfolios of both assets and liabilities, making investments in shares, bonds, and other financial assets; to fund their obligations to existing and future pensioners. The problem of selecting an investment policy for a pension fund can be analysed using asset-liability management models that allow for the non-zero correlations between the values of the assets and liabilities (e.g. rapid inflation increases the value of both equities and the liabilities of a pension scheme based on final salaries) which, if positive, reduces risk. While these problems may be formulated using quadratic programming, they have usually been solved in other ways (see Ziemba and Mulvey, 1998). Mulvey (1994) assumed that the objective was to maximise the expected value of a non-linear utility of wealth function, and specified the problem as a non-linear network problem, with the simulation of future pension fund liabilities. Similar asset-liability problems are also faced by insurance companies, for example Cariño et al (1994, 1998a, 1998b) formulated this problem for a Japanese insurance company. Their model maximises the expected market value of the company, with risk measured as the underachievement of the specified goals. It allows for various scenarios and finds solutions using stochastic linear programming. Hillier and Eckstein (1993) applied stochastic linear programming to generate the risk-return efficient frontier for a pension fund, while Holmer (1994) devised a utility maximizing approach using Monte Carlo simulation to manage the assets and liabilities of the Federal National Mortgage Association (Fannie Mae). Klaassen (1998) criticised the use of Monte Carlo simulation, as this can bias the results by including arbitrage opportunities in the sampled scenarios. To avoid this, he aggregated an arbitrage-free event tree before its inclusion in a multi-stage stochastic programming model of the asset-liability problem.

Quadratic programming has been used to form portfolios of currencies (Levy, 1981), and commercial loan portfolios (Gollinger and Morgan, 1993), in which the objective was to lend money to industrial sectors to minimise variations in industry credit quality for a given level of return (spread over LIBOR, plus fees).

Another application of quadratic programming is generalised hedging, in which the objective is usually to minimise the variance of a portfolio of a given set of assets and the chosen hedging instruments (Levy, 1987)[3]. Rudolf and Zimmermann (1998) widened this approach, and applied quadratic programming to selecting a portfolio of domestic equities, foreign equities and foreign exchange forward contracts. If the hedging instruments include options, this introduces a non-linearity into the hedging decision. Murtagh (1989), devised a non-linear programming model to hedge foreign currency exposure using a mixture of currency forward and options contracts. The aim was to minimise the expected cost, subject to a chance constraint that the probability of the cost of the chosen hedging strategy does not exceed the cost of using the forward market alone. Clewlow, Hodges and Pascoa (1998) show how linear, goal and dynamic programming can be used to hedge options in the presence of transactions costs. Nielsen and Zenios, 1996, used stochastic programming with recourse to hedge the risks of single premium deferred annuities[4] with mortgage backed securities (MBS).

Quadratic programming has also been used for constructing index tracking portfolios, where the purpose is to select a portfolio of assets (e.g. equities or bonds) which, when combined with a matching short position in the index to be tracked, has minimum risk (Meade and Salkin, 1989, 1990; Rudd, 1980; Seix and Akhoury, 1986). Multi-stage stochastic programming with recourse, in conjunction with Monte Carlo simulation to generate the scenarios, has been used by Vassiadou-Zeniou and Zenios (1996) and Zenios et al (1998) to track an index of MBS.

In some applications of portfolio theory, the decision variables must be integer. While it is acceptable to round the number of shares traded to the nearest integer, this may not be the case for futures contracts, which are generally worth over £100,000 per contract. For this reason, Peterson and Leuthold (1987) and Shanker (1993) used quadratic integer programming to compute hedging strategies involving futures. Similarly, because of indivisibilities in the quantities of short term foreign currency securities, Cotner and Levary, 1987, imposed an integer requirement when forming portfolios of currencies, making the solution procedure zero-one quadratic programming. Shapiro, 1988, used stochastic integer programming with recourse to construct bond portfolios that allow for some of the bonds being called (or redeemed early) if interest rates are low. Zero-one variables indicated whether a particular bond was included in the portfolio, while non-integer variables gave the scale of investment in each included bond.

Some authors have argued that formulation and solving quadratic programming portfolio problems is too onerous, and proposed simplified solution techniques. Sharpe (1963) proposed a single index model which simplifies the variance-covariance matrix required by the Markowitz model by assuming that assets are related to each other only through their correlation with a single common factor. This simplification removes the need for large numbers of covariance terms in the objective function, enabling the use of special purpose quadratic programming algorithms. When each asset represents only a small proportion of the portfolio, Sharpe (1967) shows that his single index model can be treated as having a linear objective function. In essence, well diversified portfolios have only systematic risk, and this is measured by asset betas, which then gives a linear objective function. In 1971, Sharpe suggested using a piecewise linear approximation to the quadratic objective function, enabling the application of linear programming to solve portfolio problems.

Another proposal is to minimize the mean absolute deviation (MAD), which can be solved using linear programming, rather than quadratic programming[5]. Konno and Yamazaki (1991 and 1997) applied MAD to forming portfolios of Japanese equities and also equities and bonds. Yawitz, Hempel and Marshall (1976) proposed viewing bond portfolios as a risk-return problem, but to avoid quadratic programming used the mean absolute price change to measure risk. The formation of portfolios of MBSs was modelled by Zenios and Kang (1993) who used both simulation and linear programming. They used Monte Carlo simulation to generate rates of return on each MBS during the holding period, and then applied linear programming to minimise the MAD, subject to the expected return exceeding some specified amount. Worzel, Vassiadou-Zeniou and Zenios (1994) suggested using both simulation and linear programming for tracking fixed-interest indices. First, the holding period returns for the securities in the index are simulated; then linear programming, with risk measured by the MAD, is used to select a portfolio which maximises the expected return, subject to the risk of underperforming the index not exceeding some specified upper bound. This bound is then minimised by iteratively solving the linear programming problem. Seix and Akhoury (1986) proposed using linear programming to devise a portfolio to track a bond index by maximising the value of the portfolio, while matching the duration[6], quality, sectors, and coupons of the bond index. Another approach to removing the need to solve a quadratic programming problem is to specify the problem as choosing between a range of pre-specified equity portfolios using data envelopment analysis (DEA) (Premachandra, Powell and Shi, 1998).

A different approach to removing the need for quadratic programming is to reformulate the portfolio problem as a non-linear generalised network model for which efficient solution algorithms exist (Mulvey, 1987). In this case the objective function is non-linear, and risk can be measured in a wide variety of ways; including the variance. Glover and Jones (1988) proposed using a network model, in conjunction with the Fourier transform, to give a linear model.

Portfolio problems, with the twin objectives of maximising returns and minimising risk, can also be viewed as goal programming problems with two goals. Additional goals can be introduced, and a number of authors have solved portfolio problems using goal programming. For example, Kumar, Philippatos and Ezzell (1978), Kumar and Philippatos (1979) and Lee and Lerro (1973) have specified models with five or six goals for forming equity portfolios. Measuring risk using systematic (beta) and unsystematic risk, rather than the variance, enables the use of linear rather than quadratic programming. Similarly, Lee, Lerro and McGinnis (1971) constructed a linear programming model with six goals for managing bond portfolios. Sharda and Musser (1986) specified a model with four goals for hedging the risk of Treasury bonds. Because they included zero-one variables to specify the week in which the hedging instrument is traded, the problem is a mixed integer goal programming problem[7].

Cheng (1962), analysed the problem of maintaining a bond portfolio over time where there is a choice between a short maturity bond with a high yield, and a longer maturity bond with a lower yield, and there is uncertainty over the available bond yields when the existing bonds mature. Cheng suggested quadratic programming by essentially treating the situation as a single period problem. Multi-period portfolio problems have been specified as dynamic programming problems (Elton and Gruber, 1971), while Mulvey and Vladimirou (1992), used a stochastic generalised network model.

In portfolio immunization the aim is to construct a portfolio of interest rate dependent securities whose value is the same as some target asset (usually another interest rate dependent asset)[8]. By matching the duration of the portfolio with that of the target asset, the portfolio is immunized against small parallel shifts in the yield curve[9]. Fong and Vasicek (1983) proposed a measure of the risk from general interest rate movements (e.g. non-parallel shifts in the yield curve) and proposed devising a bond portfolio to minimise this, subject to achieving a specified duration, by linear programming. They also suggest that the investor could compute an efficient frontier of portfolios that minimise the standard deviation of the return on the immunized portfolio for a given level of expected return, again using linear programming. Using higher moments of a generalised duration measure, Kornbluth and Salkin (1987) show it is possible to immunise against changes in the shape of the yield curve, as well as parallel shifts, using linear fractional goal programming. Nawalkha and Chambers (1996) used a modified duration measure to quantify the risk of an immunized portfolio, and minimised this using linear programming. Alexander and Resnick (1985) who incorporated default risk, also specified immunization as a linear goal programming problem. What all these immunization studies have in common is that the chosen risk measure does not involve squares or cross products of the decision variables, so that linear programming, not quadratic programming, is the solution technique.

Portfolio theory (and quadratic programming) has also been applied to problems that do not directly involve traded financial assets. For example, Freund (1956) examined farming, and Board and Sutcliffe (1991) applied the approach to tourism.

3. The Valuation of Financial Instruments

It is very important when trading in financial markets to have a good model for valuing the asset being traded, and OR techniques have made a substantial contribution in this area.

Although European style call and put options can be valued using the Black-Scholes model, which provides a good closed form solution, OR techniques have made a substantial contribution to the pricing of more complex derivatives. In 1977, Boyle proposed the use of Monte Carlo simulation as an alternative to the binomial model for pricing options for which a closed form solution is not readily available. Monte Carlo simulation has the advantage over the binomial model that its convergence rate is independent of the number of state variables (e.g. the number of underlying asset prices and interest rates), while that of the binomial model is exponential in the number of state variables.

Monte Carlo simulation is used to generate paths for the price of the underlying asset until maturity. The cash flows from the option for each path, weighted by their risk neutral probabilities[10], can then be discounted back to the present using the risk free rate, allowing the average present value across all the sample paths to be computed to give the current price of the option (Boyle, Broadie and Glasserman, 1997). A range of variance reduction methods have been used in the Monte Carlo pricing of options (e.g. control variates, antithetic variates, stratified sampling, Latin hypercube sampling, importance sampling, moment matching and conditional Monte Carlo). In addition, quasi-Monte Carlo methods have been applied to finance problems to speed up the simulation (Joy, Boyle and Tan, 1996). As well as generating option prices, Monte Carlo simulation can be used to compute the various sensitivities - "the Greeks" - including the hedge ratio, which are essential for many trading strategies (Broadie and Glasserman, 1996).

There are no closed form solutions for American style options[11], and until recently it was thought that Monte Carlo simulation could not be used to price such options. This is a major problem, as the majority of options are American style. However, progress is being made in developing Monte Carlo simulation techniques for pricing American style options (Broadie and Glasserman, 1997; Grant, Vora and Weeks, 1997). Options have also been priced using finite difference approximations, and Dempster and Hutton (1996) and Dempster, Hutton and Richards (1998) have proposed the use of linear programming to solve the finite difference approximations to the price of American style put options. In addition, American style options can be priced using dynamic programming, Dixit and Pindyck (1994).

If a closed form pricing equation cannot be derived for an option or other derivative; provided a price history is available, a neural network can be trained to produce prices using a specified set of inputs, which can then be used for out-of-sample pricing (Hutchinson, Lo and Poggio, 1994). This approach was able to outperform the Black-Scholes formula when pricing options on S&P500 futures, and has considerable potential for generating prices for "hard to price" derivatives that are already traded on competitive markets.

Empirical research has found that, although the Black-Scholes pricing model provides accurate prices for at-the-money options, there are some unexpected patterns in options prices, such as the "volatility smile"[12]. Modelling this effect, and given a contemporaneous set of prices for European style put and call options on the same underlying asset, Rubinstein (1994) has shown how the implied risk neutral probability distribution can be computed using quadratic programming. This procedure selects a set of risk neutral probabilities that minimise the sum of the squared difference between themselves and the risk neutral probabilities generated by some prior guess. These probabilities can be used to infer a recombining binomial tree that is consistent with the observed options prices, which is then used in hedging or valuing European style options on the underlying asset over the period until maturity in a way that allows for the presence of the "smile". Jackwerth and Rubinstein (1996) generalised this approach using non-linear programming to minimise four other objective functions.

Municipal authorities in the USA who wish to borrow money by issuing bonds usually invite bids from underwriting syndicates. These bids must specify a schedule of bond coupons (i.e. interest payments), subject to various restrictions imposed by the municipality, and by the need for the underwriting syndicate to market the bonds to the public. The winning bid is generally that with the lowest net interest cost to the municipality. The underwriting syndicate typically have only 15 to 30 minutes to prepare a bid, and so a computerized solution procedure is needed. This decision was formulated as a linear programming problem by Percus and Quinto (1956) and Cohen and Hammer (1965, 1966), while Weingartner (1972) respecified it as a dynamic programming problem. If the municipality places an upper bound on the number of different coupon rates, it becomes an integer programming problem, that can also be solved as a zero-one dynamic programming problem (Weingartner, 1972; Friemer, Rao and Weingartner, 1972).

Nauss and Keeler (1981) added the constraint that the coupon rates be set to integers times a specified multiplier, and proposed an integer programming formulation. The municipality can specify the true interest cost (which is the internal rate of return (IRR) on the bond), rather then the net interest cost, as the selection criterion to be used. The use of the IRR as the objective to be minimised makes the problem non-linear. Bierwag, 1976, proposed a linear programming algorithm for solving this problem. Nauss, 1986, added some additional restrictions which make the problem integer, and suggested an approximate solution using integer linear programming.

Mortgage backed securities (MBS) are created by the securitisation of a pool of mortgages. For any specific mortgage, the borrower has the right to repay the loan early - the prepayment option, or may default on the payments of capital and interest. These risks feed through to the owners of MBS, in addition to the risks of fluctuations in the rate of interest payable on flexible rate mortgages (Zipkin, 1993). Thus MBS are hybrid securities, as they are variable interest rate securities with an early exercise option. Monte Carlo simulation can be used to generate interest rate paths for future years. Forecasts of the mortgage prepayment rates then permit the computation of the cash flows from each interest rate path, and these sequences of cash flows are used to value the MBS (Zenios, 1993a; Ben-Dov, Hayre and Pica ,1992; Boyle, 1989). This procedure, which can be used to identify mispriced MBS in real time, is computationally demanding and parallel (and massively parallel) and distributed processing have been used in the solution of the problem. Simulation has also been used to price collateralised mortgage obligations or CMOs[13] (Paskov, 1997). Other hybrid securities, such as callable and putable bonds and convertible bonds face similar valuation problems to MBSs and require similarly intensive solution methods.

There is an active secondary market in loan portfolios which may carry a significant default risk. Del Angel el al (1998) used a Markov chain analysis with 14 loan performance states and Monte Carlo simulation to generate the probability distribution of the present value of loan portfolios.

4. Imperfections in Financial Markets

As well as accurately pricing financial securities, traders are interested in finding imperfections in financial markets which can be exploited to make profits (Keim and Ziemba, 1999; Ziemba, 1994). One aspect of this is the search for weak form inefficiency (i.e. that an asset's past prices can be used as the basis of a profitable trading rule). Among the early attempts to find such exploitable regularities in stock prices were Dryden's (1968, 1969) use of Markov chains.

A fundamental feature of financial markets is the existence of no-arbitrage relationships between prices, and small price discrepancies can be exploited by arbitrage trades to give large riskless profits. Network models have been used to find arbitrage opportunities between sets of currencies (Christofides, Hewins and Salkin, 1979; Kornbluth and Salkin, 1987; Mulvey, 1987; Mulvey and Vladimirou, 1992). This problem can be specified as a maximal flow network, where the aim is to maximise the flow of funds out of the network, or as a shortest path network. While some network formulations are linear and could be formulated and solved as linear programming models, interpretation of the problem as a network enables the use of computationally faster algorithms.

Chandy and Kharabe (1986) developed a model for identifying underpriced bonds. They suggested solving a linear programming model to form a bond portfolio with maximum yield. This solution then gives the break even yield, which is the minimum bond yield necessary for inclusion in the portfolio. Hodges and Schaefer (1977) devised a linear programming model which minimises the cost of a given pattern of cash flows, enabling underpriced bonds to be traded.

There has been a growing interest in using artificial intelligence based techniques (expert systems, neural networks, genetic algorithms, fuzzy logic and inductive learning) to develop trading strategies for financial markets (e.g. Trippi and Turban, 1993; Refenes, 1995; Goonatilake and Treleaven, 1995; Wong and Selvi, 1998). Such approaches have the advantage that they can pick up non-linear dynamics, and require little prior specification of the relationships involved.

Firer, Sandler and Ward, 1992, simulated the returns from a stock market timing strategy for a range of levels of forecasting skill, so quantifying the likely benefits from various levels of forecasting ability. Taylor (1989) used Monte Carlo simulation to generate a long time series of data for use in back-testing the performance of trading rules for a variety of financial assets.

5. Funding Decisions

OR techniques have also been used to help firms to determine the most appropriate method by which to raise capital from the financial markets to finance their activities. Brick, Mellon, Surkis and Mohl (1983) put forward a chance constrained linear programming model to compute the values of the debt-equity ratio each period that maximize the value of the firm. Other studies have specified the choice between various types of funding as a linear goal programming problem (Hong, 1981; Lee and Eom, 1989). Ness (1972) used linear programming to find the least cost financing decision for an investment project by a multi-national company. Kornbluth and Vinso (1982) modelled the financing decision of a multi-national corporation as involving two goals - minimizing the overall cost of capital and achieving target debt/equity ratios in each country. Since the debt/equity goals involve ratios of the decision variables, the model becomes a fractional linear goal programming problem.

A different approach to the debt problem is to assume that the firm has found its desired debt-equity ratio, and is purely concerned with raising the requisite debt as cheaply as possible. In this case, debt can be treated like any other input to the productive process, and inventory models used to determine the optimal “reorder” times and quantities (Bierman, 1966; Litzenberger and Rutenberg, 1972). An additional aspect of the problem is that, bonds’ maturity must be chosen by the borrower to reflect the different current interest rates payable on alternative maturities, the uncertain costs of future borrowing and the marketability of alternative maturities. Crane, Knoop and Pettigrew (1977) formulated this as a linear programming problem to minimise costs, which they solved for three different interest rate scenarios.

Firms, governmental organizations and others may choose to issue callable bonds in which the issuer has the option to repay the bond at a time of their choosing before the maturity date of the bond. The issuer must choose various parameters of the callable bond, and Consiglio and Zenios (1997a, 1997b) have used nonlinear programming to design such securities in a way that is most beneficial to the issuer, while Holmer, Yang and Zenios (1998) used a simulated annealing algorithm.

Firms which have issued callable debt must decide when to call (repay) the existing debt and refinance it with a new issue, presumably at a lower cost - the bond scheduling problem. This is a dynamic programming problem and has been modelled as such by Weingartner (1967), Elton and Gruber (1971) and Kraus (1973). Baker and Van Der Weide (1982) extended this model to cover a multi-subsidiary company with debt requirements for each subsidiary. Dempster and Ireland (1988) developed a model which applies a range of OR techniques in a complementary fashion to the bond scheduling problem. The model begins by using stochastic linear programming to devise a multi-period plan for both issuing and calling bonds. The plan is refined using heuristics, possibly leading to multiple plans, and the probability distributions of these revised plans are derived using simulation. Finally, an expert system is used to help in deciding between alternative plans.

An important question when appraising investment projects is determining the appropriate cost of capital, i.e. the price which must be paid in the financial markets to finance the project. Boquist and Moore (1983) proposed the use of linear goal programming to estimate the cost of capital for divisions by incorporating corporate prior beliefs concerning betas.

Certificates of deposit (CDs) are issued by banks and indicate that a specified sum has been deposited at the issuing depository institution. As such, CDs represent a source of funding for banks. Russell and Hickle (1986) developed a simulation model to predict the impact of various interest rate scenarios on the cost of this funding source.

Finally, the problem facing borrowers of choosing between alternative mortgage contracts (e.g. fixed rate, variable rate and adjustable rate mortgages) has been modelled using decision trees (Heian and Gale, 1988; Luna and Reid, 1986).

6. Strategic Problems

In recent years, some of the decisions facing traders and market makers in financial markets have been analysed using game theory (O'Hara, 1995; Dutta and Madhavan, 1997). These models typically involve one or more market makers, and traders who may be informed or uninformed, and discretionary or non-discretionary. Traders in stock markets seek to trade at the most attractive prices and large trades are often broken up into a sequence of smaller trades in an effort to minimise the price impact. This can be viewed as a strategic problem with the aim of devising a strategy for trading the block of shares. The initial trades influence the price of subsequent trades, and so executing the large trade at the lowest cost is a dynamic problem. Bertsimas and Lo (1998) use stochastic dynamic programming to define "best execution" and to compute an optimal trading strategy.

Powers (1987) applied game theory to the situation where a company has two major shareholders, and a large number of very small shareholders. This can be modelled as an oceanic game, in which the two large players behave strategically while the many small shareholders (the ocean) do not. This approach can be used to derive the highest price a large shareholder will pay in the market for corporate control.

7. *Regulatory and Legal Problems*

Financial regulators have become increasingly concerned about financial markets with their very large and rapid international financial flows. OR techniques have proved useful in regulating the capital reserves held by banks and other financial institutions to cover their risk exposure. OR techniques have also been used to ensure compliance with various legal requirements by designing appropriate strategies and to solve other legal problems relating to financial markets.

A key regulatory issue is determining the capital required by financial institutions to underpin their activities in financial markets. An increasingly popular approach to this problem is to quantify the value at risk (VAR). If the specified period and probability are 1 day and 1% respectively, then the VAR is the largest loss that will occur due to market risk 99% of the time. Thus, VAR involves quantification of the lower tail of the probability distribution of outcomes from the firm's portfolio. A particular problem with measuring risk exposure is that portfolios usually include options (or financial securities with option-like characteristics), and options have highly asymmetric payoffs. For such securities, analytical solutions to finding the probabilities in the lower tail of the payoff distribution are unreliable. Riskmetrics™ uses approximations based on "the Greeks" for options that are at or near the money[14], and Monte Carlo simulation for other options positions (Morgan and Reuters, 1996). Monte Carlo simulation can either make distributional assumptions, or use the distribution of historical realizations, i.e. bootstrapping (Pritsker, 1997)[15].

While Riskmetrics™ quantifies market risk, some securities are also subject to credit risk. Although the market risk of financial instruments (apart from options) tends to produce returns with an approximately normal distribution, credit risk produces returns that are highly non-normal for all instruments. Usually there is no default, while occasionally there is a substantial or total default. Therefore, Monte Carlo simulation is relevant to modelling the credit risk of portfolios of financial instruments (e.g. loans, letters of credit, bonds, trade credit, swaps, forwards) as in CreditMetrics™ (Morgan, 1997).

Data envelopment analysis (DEA) has been used to assist in bank regulation by measuring bank efficiency, which is then used to predict bank failure (Barr, Seiford and Siems, 1993; Bauer, Berger, Ferrier and Humphrey, 1998).

Traders are required to put up margin when they trade options, and there are complicated rules for determining the total margin required on a portfolio of options and shares. Traders wish to minimise their margin payments, and Rudd and Schroeder (1982) have developed a linear programming model in which the problem was modelled as a transportation problem for determining the minimum required margin.

Some MBS are traded on a "to-be-announced" basis with forward delivery. In these cases the originators have mortgages that have not yet been pooled, and this gives them some flexibility in structuring the securitisation in a manner beneficial to themselves. An extensive set of rules governs the way in which a "to-be-announced" MBS can be structured, leading to a complex problem in devising a feasible solution. This can be specified as a complicated integer programming problem (with the objective of maximising the originator's profit). Collateralised mortgage obligations (CMOs) also involve the securitisation of a mortgage pool, but in this case the pool is structured into a series of bonds (or tranches), each with a different maturity and risks. In the USA various legal restrictions apply to how CMOs can be structured, and it may be difficult to find a feasible solution. Dahl, Meeraus and Zenios (1993) have proposed a complex zero-one programming model for solving this problem, with the objective of maximizing the proceeds from the issue.

To receive the tax benefits, leveraged leases in the USA are designed to satisfy the Internal Revenue Service rules. Capettini and Toole (1981) proposed an integer programming model to structure leveraged leases to meet the IRS rules, with the objective of maximising the net present value of the lessor's cash flow. Litty (1994) developed an approach to this problem using linear programming heuristics that provided fast solutions for untrained users.

Sharda (1987) proposed a linear programming formulation to establish the maximum loss that investors could have sustained from trading in a company's shares. This figure can then be used by the company's lawyers when fighting a lawsuit claiming damages from a misleading statement by the company.

In August 1982 the Kuwait Stock Market collapsed leaving $94 billion of debt to be resolved. This led to the problem of devising a fair method for distributing the assets seized from insolvent brokers among the other brokers and private investors. This problem was solved using linear programming, which reduced the total unresolved debt to $20 billion, saving an estimated $10.34 billion in lawyer's fees (Taha, 1991, Elimam, Girgis and Kotob, 1996, 1997).

8. Economic Understanding

In addition to its traditional role of improving the quality of decision making, OR can also help in trying to understand the economic forces shaping the finance sector. Financial innovation may occur when there is an exogenous change in the constraints or in the costs of meeting existing constraints. Using a linear programming model of a bank, Ben-Horim and Silber (1977) employed annual data to compute movements in the shadow prices of the various constraints. They suggested that a rise in the shadow price of the deposits constraint led to the financial innovation of negotiable CDs.

Arbitrage Pricing Theory (APT), which can be viewed as a generalization of the Capital Asset Pricing Model (CAPM), seeks to identify the factors which affect asset returns. Most tests of the APT use factor analysis, and have difficulty in determining the number and definition of the factors that influence asset returns. To overcome these problems Ahmadi (1993) suggested using a neural network to test the APT. This also has the advantage that the results are distribution free.

9. Conclusions

Mathematical programming is the OR technique that has been most widely applied in financial markets. Most types of mathematical programming have been employed - linear, quadratic, non-linear, integer, goal, chance constrained, stochastic, fractional, DEA and dynamic. Mathematical programming has been used to solve a considerable range of problems in financial markets - forming portfolios of equities, bonds, loans and currencies, generalized hedging, immunization, equity and bond index tracking, estimating the implied risk neutral probabilities for options, devising a schedule of coupons for municipal bond bids, identifying underpriced bonds, setting the firm's debt-equity ratio, deciding when to refinance outstanding bonds, estimating the divisional cost of capital, determining the required minimum option margin, structuring MBS and CMO securitisations, creating a trading strategy to execute a block trade, designing leveraged leases, computing the maximum loss sustained by shareholders, spotting insolvent banks, sorting out the failure of a stock exchange and understanding the forces leading to financial innovations.

Monte Carlo simulation is also widely used in financial markets - mainly to value exotic options and securities with embedded options, and to estimate the VAR for various financial institutions. Simulation has also been useful in testing trading rules, and for examining the risks of a position in securities. In some cases the use of OR techniques has influenced the way financial markets function since they permit traders to make better decisions in less time. For example, exotic options would trade with much wider bid-ask spreads, if they traded at all, in the absence of the accurate prices computed using Monte Carlo simulation.

Other OR techniques are less used in financial markets. Arbitrage and multi-period portfolio problems have been formulated as network models, while market efficiency has been tested using neural networks. Game theory has been applied to battles for corporate control, decision trees to analyse mortgage choice, inventory models to set the size and timing of corporate bond issues, and Markov chains to valuing loan portfolios and testing market efficiency. One important OR technique has found little application in financial markets - queuing theory.

The main areas of financial markets in which OR techniques have been applied are portfolio problems and pricing complex financial instruments accurately. OR techniques can also be used by financial regulators and financial institutions in setting capital adequacy standards. Some other application areas also exist - devising feasible solutions that meet a complicated set of the legal requirements, making funding decisions, spotting imperfections and arbitrage opportunities in financial markets and solving strategic problems.

The relationship between finance and OR is bidirectional. Not only have various OR techniques been applied to finance problems, but finance theories have created a need to develop and improve OR solution techniques, and at least two Nobel prize winners in finance have made contributions to OR. Markowitz was honoured in 1989 by ORSA/TIMS for his work on sparse matrices and inventing the computer simulation language SIMSCRIPT; while both he and Sharpe have produced computer algorithms for solving portfolio problems[16].

This paper indicates that OR techniques play an important role in financial markets and, with the recent dramatic improvements in the real time availability of data and in computer speed, this role will increase. This will create the opportunity for OR techniques to play an even greater role in financial markets.

Endnotes

1. If there are no such inequality constraints the problem can be solved using classical optimization techniques, and quadratic programming is unnecessary.

2. MBS are considered in more detail in section 3 below.

3. Quadratic programming is helpful in solving generalized hedging problems in which a number of assets is hedged using a variety of instruments. If a single asset is to be hedged using one or more hedging instruments, the risk minimising hedge ratios can be obtained straightforwardly without the need for mathematical programing (Anderson and Danthine, 1981).

4. The buyer of a single premium deferred annuity (SPDA) makes a single payment to the insurer. The insurer then pays interest into this amount until the buyer retires, after which time the insurer pays the buyer an annuity. Before retirement the buyer has the option to withdraw part or all of the money in their account.

5. Under joint normality of asset returns, both mean-variance and MAD give the same efficient set.

6. Duration is the interest rate elasticity of the bond price, and is used as a measure of bond portfolio interest rate risk.

7. As noted above, with one asset to be hedged using a single hedging instrument, the risk minimizing hedge ratio can be computed directly without the need for quadratic programming.

8. There is also a literature on managing the assets and liabilities held by banks (which are taken to exclude equities), where the objective is usually to maximise the value (or expected value) of the portfolio over one (or many) time periods (net of penalty costs from constraint target violations), subject to restrictions of the total investment, maximum capital loss and various bank regulations (e.g. Cohen and Hammer, 1967; Chambers and Charnes, 1961; Kusy and Ziemba, 1986; Hiller and Eckstein, 1993). These studies are closely related to cash management models (Kallberg, White and Ziemba, 1982). A survey of current research in this area is Ziemba and Mulvey (1998).

9. The yield curve shows the interest rates for different maturities.

10. Risk neutral probabilities are the probabilities which can be inferred from prices by assuming that investors are risk neutral.

11. American style options can be exercised at any time before the option expires.

12. A smile occurs when the implied volatility for deep in-the-money and deep out-the-money options exceeds that for at-the-money options, i.e. the relationship between the

strike price and implied volatility is U-shaped.

13. CMOs are considered further in section 7.

14. An option is at-the-money occurs when the current price of the underlying asset is close to the price at which the option can be exercised.

15. A related application of Monte Carlo simulation is stress testing, which quantifies the sensitivity of a portfolio to specified, often adverse, market scenarios.

16. William Sharpe was a member of the Logistics Department at the Rand Corporation where he wrote a paper on optimizing the design of military transport aircraft, and his job title at the University of Washington was Associate Professor of Operations Research.

References

Ahmadi, H. (1993) Testability of the Arbitrage Pricing Theory by Neural Networks. In *Neural Networks in Finance and Investing: Using Artificial Intelligence to Improve Real World Performance*, edited by R.R. Trippi, R.R. and E. Turban, Probus Publishing Co, Chicago, pp. 421-432.

Alexander, G.J. and Resnick, B.G. (1985) Using Linear and Goal Programming to Immunize Bond Portfolios, *Journal of Banking and Finance*, vol. 9, no. 1, March, pp. 35-54.

Anderson, R.W. and Danthine, J.P. (1981) Cross Hedging, *Journal of Political Economy*, vol. 89, no. 6, December, pp. 1182-1196.

Ashford, R.W., Berry, R.H. and Dyson, R.G. (1988) Operational Research and Financial Management, *European Journal of Operations Research*, vol. 36, no. 2, pp. 143-152.

Baker, K.R. and Van Der Weide, J.H. (1982) The Bond Scheduling Problem of the Multi-Subsidiary Holding Company, *Management Science*, vol. 28, no. 7, July, pp. 738-748.

Barr, R.S., Seiford, L.M. and Siems, T.F. (1993) An Envelopment Analysis Approach to Measuring the Managerial Efficiency of Banks, *Annals of Operations Research*, vol. 45, nos. 1/4, December, pp. 1-19.

Bauer, P.W., Berger, A.N., Ferrier, G.D. and Humphrey, D.B. (1998) Consistency Conditions for Regulatory Analysis of Financial Institutions: A Comparison of Frontier Efficiency Methods, *Journal of Economics and Business*, vol. 50, no. 2, pp. 85-114.

Ben-Dov, Y., Hayre, L. and Pica, V. (1992) Mortgage Valuation Models at Prudential Securities, *Interfaces*, vol. 22, no. 1, January-February, pp. 55-71.

Ben-Horim, M. and Silber, W.L. (1977) Financial Innovation: A Linear Programming Approach, *Journal of Banking and Finance*, vol. 1, no. 3, November, pp. 277-296.

Bernstein, P.L. (1992) *Capital Ideas: The Improbable Origins of Modern Wall Street*, The Free Press, Macmillan Inc, New York.

Bertsimas, D. and Lo, A.W. (1998) Optimal Control of Execution Costs, *Journal of Financial Markets*, vol. 1, no. 1, April, pp. 1-50.

Bierman, H. (1966) The Bond Size Decision, *Journal of Financial and Quantitative Analysis*, vol. 1, no. 4, December, pp. 1-14.

Bierwag, G.O. (1976) Optimal TIC Bids on Serial Bond Issues, *Management Science*, vol. 22, no. 11, July, pp. 1175-1185.

Board, J.L.G. and Sutcliffe, C.M.S. (1991) Risk and Income Tradeoffs in Regional Policy: A Portfolio Theoretic Approach, *Journal of Regional Science*, Vol 31, No 2, April 1991, pp 191-210.

Board, J.L.G. and Sutcliffe, C.M.S. (1994) Estimation Methods in Portfolio Selection and the Effectiveness of Short Sales Restrictions: UK Evidence, *Management Science*, Vol. 40, No. 4, April 1994, pp. 516-534.

Board, J.L.G., Sutcliffe, C.M.S. and Ziemba, W.T. (1999) Portfolio Theory: Mean Variance. In *Encyclopaedia of Operations Research and Management Science* edited by Saul I. Gass and Carl M. Harris, Kluwer Academic Publishing, Boston-Dordrecht-London, forthcoming.

Boquist, J.A. and Moore, W.T. (1983) Estimating the Systematic Risk of an Industry Segment: A Mathematical Programming Approach, *Financial Management*, vol. 12, no. 4, Winter, pp. 11-18.

Boyle, P.P. (1977) Options: A Monte Carlo Approach, *Journal of Financial Economics*, vol. 4, no. 3, May, pp. 323-338.

Boyle, P.P. (1989) Valuing Canadian Mortgage Backed Securities, *Financial Analysts Journal,* vol. 45, no. 3, May-June, pp. 55-60.
Boyle, P.P., Broadie, M. and Glasserman, P. (1997) Monte Carlo Methods for Security Pricing, *Journal of Economic Dynamics and Control*, vol. 21, nos. 8-9, June, pp. 1267-1321.
Bradley, S.P. and Crane, D.B. (1972) A Dynamic Model for Bond Portfolio Management, *Management Science*, vol. 19, no. 2, October, pp. 139-151.
Brick, I.E., Mellon, W.G., Surkis, J. and Mohl, M. (1983) Optimal Capital Structure: A Multi Period Programming Model for Use in Financial Planning, *Journal of Banking and Finance*, vol. 7, no. 1, March, pp. 45-67.
Broadie, M. and Glasserman, P. (1996) Estimating Security Price Derivatives Using Simulation, *Management Science*, vol. 42, no. 2, February, pp. 269-285.
Broadie, M. and Glasserman, P. (1997) Pricing American Style Securities Using Simulation, *Journal of Economic Dynamics and Control*, vol. 21, nos. 8-9, June, pp. 1323-1352.
Cariño, D.R., Kent, T., Myers, D.H., Stacy, C., Sylvanus, M., Turner, A.L., Watanabe, K. and Ziemba, W.T. (1994) The Russell-Yasuda Kasai Model: An Asset-Liability Model for a Japanese Insurance Company Using Multistage Stochastic Programming, *Interfaces,* vol. 24, no. 1, January-February, pp. 29-49. Reprinted in Ziemba and Mulvey (1998)
Cariño, D.R. and Ziemba, W.T. (1998a) Formulation of the Russell Yasuda Kasai Financial Planning Model, *Operations Research*, vol. 46, no. 4, July-August, pp. 433-449.
Cariño, D.R., Myers, D. and Ziemba, W.T. (1998b) Concepts, Technical Issues and Uses of the Russell Yasuda Kasai Model, *Operations Research*, vol. 46, no. 4, July-August, pp. 450-462.
Capettini, R. and Toole, H. (1981) Designing Leveraged Leases: A Mixed Integer Linear Programming Approach, *Financial Management*, vol. 10, no. 4, Autumn, pp. 15-23.
Chambers, D. and Charnes, A. (1961) Intertemporal Analysis and Optimization of Bank Portfolios, *Management Science*, vol. 7, no. 4, July, pp. 393-410.
Chandy, P.R. and Kharabe, P. (1986) Pricing in the Government Bond Market, *Interfaces,* vol. 16, no. 5, September-October, pp. 65-71.
Cheng, P.L. (1962) Optimum Bond Portfolio Selection, *Management Science*, vol. 8, no. 4, July, pp. 490-499.
Christofides, N., Hewins, R.D. and Salkin, G.R. (1979) Graph Theoretic Approaches to Foreign Exchange, *Journal of Financial and Quantitative Analysis*, vol. 14, no. 3, September, pp. 481-500.
Clewlow, L., Hodges, S. and Pascoa, A. (1998) Mathematical Programming and Risk Management of Derivative Securities. In *Operational Tools in the Management of Financial Risks*, edited by C. Zopounidis, Kluwer Academic Publishers, 1998, pp. 237-248.
Cohen, K.J. and Hammer, F.S. (1965) Optimal Coupon Schedules for Municipal Bonds, *Management Science*, vol. 12, no. 1, September, pp. 68-82.
Cohen, K.J. and Hammer, F.S. (1966) Optimal Level Debt Schedules for Municipal Bonds, *Management Science*, vol. 13, no. 3, November, pp. 161-166.
Cohen, K.J. and Hammer, F.S. (1967) Linear Programming and Optimal Bank Asset Management Decisions, *Journal of Finance*, vol. 22, no. 2, May, pp. 147-168.
Consiglio, A. and Zenios, S.A. (1997a) A Model for Designing Callable Bonds and its Solution Using Tabu Search, *Journal of Economic Dynamics and Control*, vol. 21, no. 8-9, June, pp. 1445-1470.
Consiglio, A. and Zenios, S.A. (1997b) High Performance Computing for the Computer Aided

Design of Financial Products. In *Advances in High Performance Computing* edited by L. Grandinetti, J. Kowalik and M. Vajtersic, NATO Advanced Science Institute Series, vol 30, Kluwer Academic Publishers, pp. 273-302.

Cotner, J.S. and Levary, R.R. (1987) A Quadratic Programming Model for Determining Short Term Multiple Currency Portfolios, *OPSEARCH*, vol. 24. No. 4, pp. 218-227.

Crane, D.B., Knoop, F. and Pettigrew, W. (1977) An Application of Management Science to Bank Borrowing Strategies, *Interfaces*, vol. 8, no. 1, part 2, November, pp. 70-81.

Dahl, H., Meeraus, A. and Zenios, S.A. (1993) Some Financial Optimization Models: II Financial Engineering. In *Financial Optimization*, edited by S.A. Zenios, Cambridge University Press, pp. 37-71.

Del Angel, G.F., Márquez and Patiño, E.P. (1998) A Discrete Markov Chain Model for Valuing Loan Portfolios. The Case of Mexican Loan Sales, *Journal of Banking and Finance*, vol. 22, no. 10-11, October, pp. 1457-1480.

Dempster, M.A.H. and Hutton, J.P. (1996) Pricing American Stock Options by Linear Programming, Working Paper, Department of Mathematics, University of Essex, October, 34 pages.

Dempster, M.A.H., Hutton, J.P. and Richards, D.G. (1998) LP Valuation of Exotic American Options Exploiting Structure, Working paper, Judge Institute of Management Studies, University of Cambridge, WP 27/98, September.

Dempster, M.A.H. and Ireland, A.M. (1988) A Financial Expert Decision Support System. In *Mathematical Models for Decision Support*, edited by G. Mitra, NATO ASI Series, vol. F48, Springer-Verlag, pp. 415-440.

Dixit, A.K. and Pindyck, R.S. (1994) *Investment Under Uncertainty*, Princeton University Press, New Jersey.

Dryden, M.M. (1968) Short-Term Forecasting of Share Prices: An Information Theory Approach, *Scottish Journal of Political Economy*, vol. 15, November, pp 227-249.

Dryden, M.M. (1969) Share Price Movements: A Markovian Approach, *Journal of Finance*, vol 24, No 1, March, pp 49-60.

Dutta, P.K. and Madhavan, A. (1997) Competition and Collusion in Dealer Markets, *Journal of Finance*, vol. 52, no. 1, March, pp. 245-276.

Elimam, A.A., Girgis, M and Kotob, S. (1996) The Use of Linear Programming in Disentangling the Bankruptcies of Al-Manakh Stock Market Crash, *Operations Research*, vol. 44, no. 5, September-October, pp. 665-676.

Elimam, A.A., Girgis, M and Kotob, S. (1997) A Solution to Post Crash Debt Entanglements in Kuwait's al-Manakh Stock Market, *Interfaces*, vol. 27, no. 1, January-February, pp. 89-106.

Elton, E.J. and Gruber, M.J. (1971) Dynamic Programming Applications in Finance, *Journal of Finance*, vol. 26, no. 2, May, pp. 473-506.

Firer, C., Sandler, M. and Ward, M. (1992) Market Timing: A Worthwhile Strategy?, *Omega*, vol. 20, no. 3, pp. 313-322.

Fong, H.G. and Vasicek, O. (1983) The Tradeoff Between Return and Risk in Immunized Portfolios, *Financial Analysts Journal*, vol. 39, no. 5, September-October, pp. 73-78.

Friemer, M., Rao, M.R. and Weingartner, H.M. (1972) Note on "Municipal Bond Coupon Schedules With Limitations on the Number of Coupons", *Management Science*, vol. 19, no. 4, December, pp. 379-380.

Freund, R.J. (1956) The Introduction of Risk in a Programming Model, *Econometrica*, vol. 24, no. 3. July, pp. 253-263.

Glover, F. and Jones, C.K. (1988) A Stochastic Generalized Network Model and Large Scale Mean-Variance Algorithm for Portfolio Selection, *Journal of Information and Optimization Sciences,* vol. 9, no. 3, pp. 299-316.

Gollinger, T.L. and Morgan, J.B. (1993) Calculation of an Efficient Frontier for a Commercial

Loan Portfolio, *Journal of Portfolio Management*, vol. 19, no. 2, Winter, pp. 39-46.

Golub, B., Holmer, M., McKendall, R., Pohlman, L. and Zenios, S.A. (1995) A Stochastic Programming Model for Money Management, *European Journal of Operational Research*, vol. 85, pp. 282-296.

Goonatilake, S. and Treleaven, P. (editors) (1995) *Intelligent Systems for Finance and Business*, John Wiley and Sons.

Grant, D,. Vora, G. and Weeks, D. (1997) Path Dependent Options: Extending the Monte Carlo Simulation Approach, *Management Science*, vol. 43, no. 11, November, pp. 1589-1602.

Heian, B.C. and Gale, J.R. (1988) Mortgage Selection Using a Decision Tree Approach: An Extension, *Interfaces*, vol. 18, no. 4, July-August, pp. 72-81.

Hillier, R.S. and Eckstein, J. (1993) Stochastic Dedication: Designing Fixed Income Portfolios Using Massively Parallel Benders Decomposition, *Management Science*, vol. 39, no. 11, November, pp. 1422-1438.

Hodges, S.D. and Schaefer, S.M. (1977) A Model for Bond Portfolio Improvement, *Journal of Financial and Quantitative Analysis*, vol. 12, no. 2, June, pp. 243-260.

Holmer, M.R. (1994) The Asset-Liability Management Strategy System at Fannie Mae, *Interfaces*, vol. 24. no. 3, May-June, pp. 3-21.

Holmer, M.R., Yang, D. and Zenios, S.A. (1998) Designing Callable Bonds Using Simulated Annealing. In *Operational Tools in the Management of Financial Risks*, edited by C. Zopounidis, Kluwer Academic Publishers, 1998, pp. 177-196.

Hong, H.K. (1981) Finance Mix and Capital Structure, *Journal of Business Finance and Accounting*, vol. 8, no. 4, Winter, pp. 485-491.

Hutchinson, J.M., Lo, A.W. and Poggio, T. (1994) A Non-Parametric Approach to Pricing and Hedging Derivative Securities Via Learning Networks, *Journal of Finance*, vol. 49, no. 3, July, pp. 851-889.

Jackwerth, J.C. and Rubinstein, M, (1996) Recovering Probability Distributions from Options Prices, *Journal of Finance*, vol. 51, no. 5, December, pp. 1611-1631.

Joy, C., Boyle, P.P. and Tan, K.S. (1996) Quasi-Monte Carlo Methods in Numerical Finance, *Management Science*, vol. 42, no. 6, June, pp. 926-938.

Kallberg, J.G., White, R. and Ziemba, W.T. (1982) Short Run Financial Planing Under Uncertainty, *Management Science*, vol. 28, no. 6, June, pp. 670-682.

Keim, D.B. and Ziemba, W.T. eds. (1999) *Security Market Imperfections in Worldwide Equity Markets*, Cambridge University Press.

Klaassen, P. (1998) Financial Asset-Pricing Theory and Stochastic Programming Models for Asset/Liability Management: A Synthesis, *Management Science*, vol. 44, no. 1, January, pp. 31-48.

Konno, H. and Yamazaki, H. (1991) Mean Absolute Deviation Portfolio Optimization Model and its Applications to Tokyo Stock Market, *Management Science*, vol. 37, no. 5, May, pp. 519-531.

Konno, H. and Yamazaki, H. (1997) An Integrated Stock-Bond Portfolio Optimization Model, *Journal of Economic Dynamics and Control*, vol. 21, nos. 8-9, June, pp. 1427-1444.

Kornbluth, J.S.H. and Vinso, J.D. (1982) Capital Structure and the Financing of the Multinational Corporation: A Fractional Multi-Objective Approach, *Journal of Financial and Quantitative Analysis*, vol. 17, no. 2, June, pp. 147-178.

Kornbluth, J.S.H. and Salkin, G.R. (1987) *The Management of Corporate Financial Assets: Applications of Mathematical Programming Models*, Academic Press Inc, London.

Kraus, A. (1973) The Bond Refunding Decision in an Efficient Market, *Journal of Financial and Quantitative Analysis*, vo. 8, no. 5, December, pp. 793-806.

Kumar, P.C. and Philippatos, G.C. (1979) Conflict Resolution in Investment Decisions: Implementation of Goal Programming Methodology for Dual Purpose Funds, *Decision Sciences*, vol. 10, October, pp. 562-576.

Kumar, P.C., Philippatos, G.C. and Ezzell, J.R. (1978) Goal Programming and the Selection of Portfolios by Dual Purpose Funds, *Journal of Finance*, vol. 33, no. 1, March, pp. 303-310.
Kusy, M.I. and Ziemba, W.T. (1986) A Bank Asset and Liability Management Model, *Operations Research,* vol. 34, no. 3, May-June, pp. 356-376.
Lee, S.M. and Eom, H.B. (1989) A Multi Criteria Approach to Formulating International Project Financing Strategies, *Journal of the Operational Research Society*, vol. 40, no. 6, June, pp. 519-528.
Lee, S.M. and Lerro, A.J. (1973) Optimizing the Portfolio Selection for Mutual Funds, *Journal of Finance*, vol. 28, no. 5, December, pp. 1087-1101.
Lee, S.M., Lerro, A.J .and McGinnis, B.R. (1971) Optimization of Tax Switching for Commercial Banks, *Journal of Money, Credit and Banking*, vol. 3, May (part 1), pp. 293-303.
Levy, H. (1981) Optimal Portfolio of Foreign Currencies with Borrowing and Lending, *Journal of Money Credit and Banking*, vol. 13, no. 3, August, pp. 325-341.
Levy, H. (1987) Futures, Spots, Stocks and Bonds: Multi-Asset Portfolio Analysis, *Journal of Futures Markets*, vol. 7, no. 4, August, pp. 383-395.
Litty, C.J. (1994) Optimal Lease Structuring at GE, *Interfaces,* vol. 24, no. 3, May-June, pp. 34-45.
Litzenberger, R.H. and Rutenberg, D.P. (1972) Size and Timing of Corporate Bond Flotations, *Journal of Financial and Quantitative Analysis*, vol. 7, no. 1, January, pp. 1343-1359.
Luna, R.E. and Reid, R.A. (1986) Mortgage Selection Using a Decision Tree Approach, *Interfaces*, vol. 16, no. 3, May-June, pp. 73-81.
Markowitz, H. (1952) Portfolio Selection, *Journal of Finance*, vol. 7, no. 1, March, pp. 77-91.
Markowitz, H. (1987) *Mean-Variance in Portfolio Choice and Capital Markets*, Blackwell.
Meade, N. and Salkin, G.R. (1989) Index Funds - Construction and Performance Measurement, *Journal of the Operational Research Society*, vol. 40, no. 10, October, pp. 871-879.
Meade, N. and Salkin, G.R. (1990) Developing and Maintaining an Equity Index Fund, *Journal of the Operational Research Society*, vol. 41, no. 7, July, pp. 599-607.
Merton, R.C.(1995) Influence of Mathematical Models in Finance on Practice: Past, Present and Future. In *Mathematical Models in Finance* edited by S.D. Howison, F.P. Kelly and P. Wilmott, Chapman and Hall, London, pp. 1-13.
Morgan, J.P. and Reuters (1996) *RiskMetrics™ - Technical Document*, Morgan Guarantee Trust Company of New York.
Morgan, J.P. (1997) *CreditMetrics™ - Technical Document*, Morgan Guarantee Trust Company of New York.
Mulvey, J.M. (1987) Nonlinear Network Models in Finance. In *Advances in Mathematical Programming and Financial Planning* edited by K.D. Lawrence, J.B. Guerard and G.R. Reeves, vol. 1, JAI Press, pp. 253-271.
Mulvey, J.M. (1994) An Asset Liability Investment System, *Interfaces,* vol. 24, no. 3, May-June, pp. 22-33.
Mulvey, J.M. and Vladimirou, H. (1992) Stochastic Network Programming for Financial Planning Problems, *Management Science*, vol. 38, no. 11, November, pp. 1642-1664.
Murtagh, B.A. (1989) Optimal Use of Currency Options, *Omega*, vol. 17, no. 2, pp. 189-192.
Nauss, R.M. (1986) True Interest Cost in Municipal Bond Bidding: An Integer Programming Approach, *Management Science*, vol. 32, no. 7, July, pp. 870-877.
Nauss, R.M. and Keeler, B.R. (1981) Minimizing Net Interest Cost in Municipal Bond Bidding, *Management Science*, vol. 27, no. 4, April, pp. 365-376.
Nawalkha, S.K. and Chambers, D.R. (1996) An Improved Immunization Strategy: M-Absolute, *Financial Analysts Journal*, vol. 52, no. 5, September-October, pp. 69-76.
Ness, W.L. (1972) A Linear Programming Approach to Financing the Multinational Corporation,

Financial Management, vol. 1, no. 3, Winter, pp. 88-100.
Nielsen, S.S. and Zenios, S.A. (1996) A Stochastic Model for Funding Single Premium Deferred Annuities, *Mathematical Programming*, vol. 75, no. 2, pp. 177-200.
O'Hara, M. (1995) *Market Microstructure Theory*, Blackwell Business.
Paskov, S.H. (1997) New Methodologies for Valuing Derivatives. In *Mathematics of Derivative Securities* edited by Michael A.H. Dempster and S.R. Pliska, Cambridge University Press, pp. 545-582.
Perold, A.F. (1984) Large Scale Portfolio Optimization, *Management Science*, vol. 30, no. 10, October, pp. 1143-1160.
Percus, J. and Quinto, L. (1956) The Application of Linear Programming to Competitive Bond Bidding, *Econometrica,* vol. 24, no. 4, October, pp. 413-428.
Peterson, P.E. and Leuthold, R.M. (1987) A Portfolio Approach to Optimal Hedging for a Commercial Cattle Feedlot, *Journal of Futures Markets*, vol. 7, no. 4, August, pp. 443-457.
Powers, I.Y. (1987) A Game Theoretic Model of Corporate Takeovers by Major Stockholders, *Management Science*, vol. 33, no. 4, April, pp. 467-483.
Premachandra, I., Powell, J.G. and Shi, J. (1998) Measuring the Relative Efficiency of Fund Management Strategies in New Zealand Using a Spreadsheet-based Stochastic Data Envelopment Analysis Model, *Omega*, vol. 26, no. 2, May, pp. 319-331.
Pritsker, M. (1997) Evaluating Value at Risk Methodologies: Accuracy Versus Computational Time, *Journal of Financial Services Research*, vol. 12, nos 2-3, pp. 201-242.
Refenes, A.P. (editor) (1995) *Neural Networks in the Capital Markets*, John Wiley and Sons.
Rubinstein, M. (1994) Implied Binomial Trees, *Journal of Finance,* vol. 49, no. 3, July, pp. 771-818.
Rudd, A. (1980) Optimal Selection of Passive Portfolios, *Financial Management*, vol. 9, no. 1, Spring, pp. 57-66.
Rudd, A. and Schroeder, M. (1982) The Calculation of Minimum Margin, *Management Science*, vol. 28, no. 12, December, pp. 1368-1379.
Rudolf, M. and Zimmermann, H. (1998) An Algorithm for International Portfolio Selection and Optimal Currency Hedging. Printed in Ziemba and Mulvey (1998), pp. 315-340.
Russell, R.A. and Hickle, R. (1986) Simulation of a CD Portfolio, *Interfaces,* vol. 16, no. 3, May-June, pp. 49-54.
Seix, C. and Akhoury, R. (1986) Bond Indexation: The Optimal Quantitative Approach, *Journal of Portfolio Management*, vol. 12, no. 3, Spring, pp. 50-53.
Shanker, L. (1993) Optimal Hedging Under Indivisible Choices, *Journal of Futures Markets*, vol. 13, no. 3, May, pp. 237-259.
Shapiro, J.F. (1988) Stochastic Programming Models for Dedicated Portfolio Selection. In *Mathematical Models for Decision Support*, edited by G. Mitra, NATO ASI Series, vol. F48, Springer-Verlag, pp. 587-611.
Sharda, R. (1987) A Simple Model to Estimate Bounds on Total Market Gains and Losses for a Particular Stock, *Interfaces*, vol. 17, no. 5, September-October, pp. 43-50.
Sharda, R. and Musser, K.D. (1986) Financial Futures Hedging Via Goal Programming, *Management Science*, vol. 32, no. 8, August, pp. 933-947.
Sharpe, W.F. (1963) A Simplified Model for Portfolio Analysis, *Management Science*, vol. 9, no. 1, January, pp. 277-293.
Sharpe, W.F. (1967) A Linear Programming Algorithm for Mutual Fund Portfolio Selection, *Management Science*, vol. 13, no. 7, March, pp. 499-510.
Sharpe, W.F. (1971) A Linear Programming Approximation for the General Portfolio Analysis Problem, *Journal of Financial and Quantitative Analysis*, vol. 6, no. 5, December, pp. 1263-1275.
Taha, H.A. (1991) Operations Research Analysis of a Stock Market Problem, *Computers and*

Operations Research, vol. 18, no. 7, pp. 597-602.
Taylor, S.J. (1989) Simulating Financial Prices, *Journal of the Operational Research Society*, vol. 40, no. 6, June, pp. 567-569.
Trippi, R.R. and Turban, E. (editors) (1993) *Neural Networks in Finance and Investing: Using Artificial Intelligence to Improve Real World Performance*, Probus Publishing Co, Chicago.
Vassiadou-Zeniou, C. and Zenios, S.A. (1996) Robust Optimization Models for Managing Callable Bond Portfolios, *European Journal of Operational Research*, vol. 91, pp. 264-273.
Weingartner, H.M. (1967) Optimal Timing of Bond Refunding, *Management Science*, vol. 13, no. 7, March, pp. 511-524.
Weingartner, H.M. (1972) Municipal Bond Coupon Schedules With Limitations on the Number of Coupons, *Management Science*, vol. 19, no. 4, December, pp. 369-378.
Wong, K. and Selvi, Y. (1998) Neural Network Applications in Finance: A Review and Analysis of Literature (1990-1996), *Information and Management*, vol. 34, no. 3, pp. 129-139.
Worzel, K.J., Vassiadou-Zeniou, C. and Zenios, S.A. (1994) Integrated Simulation and Optimization Models for Tracking Indices of Fixed Income Securities, *Management Science*, vol. 42, no. 2, March-April, pp. 223-233.
Yawitz, J.B., Hempel, G.H. and Marshall, W.J. (1976) A Risk-Return Approach to the Selection of Optimal Government Bond Portfolios, *Financial Management*, vol. 5, no. 3, Autumn, pp. 36-45.
Zenios, S.A. (1991) Massively Parallel Computations for Financial Planning Under Uncertainty. In *Very Large Scale Computation in the 21st Century*, edited by Jill P. Mesirov, Society for Industrial and Applied Mathematics, Philadelphia, pp. 273-294.
Zenios, S.A. (1993a) Parallel Monte Carlo Simulation of Mortgage Backed Securities. In *Financial Optimization*, edited by S.A. Zenios, Cambridge University Press., pp. 325-343.
Zenios, S.A. (1993b) A Model for Portfolio Management with Mortgage Backed Securities, *Annals of Operations Research*, vol. 43, no. 1-4, pp. 337-356.
Zenios, S.A., Holmer, M.R., McKendall, R. and Vassiadou-Zeniou, C. (1998) Dynamic Models for Fixed Income Portfolio Management Under Uncertainty, *Journal of Economic Dynamics and Control*, vol. 22. no. 10, September, pp. 1517-1541.
Zenios, S.A. and Kang, P. (1993) Mean Absolute Deviation Portfolio Optimization for Mortgage Backed Securities, *Annals of Operations Research*, vol. 45, nos. 1-4, December, pp. 433-450.
Ziemba, W.T. (1994), World Wide Security Market Regularities, *European Journal of Operational Research*, vol. 74, no. 2, 198-229.
Ziemba, W.T. and Mulvey, J.M. eds. (1998) *Worldwide Asset and Liability Modelling*, Cambridge University Press.
Zipkin, P. (1993) Mortgages and Markov Chains: A Simplified Evaluation Model, *Management Science*, vol. 39, no. 6, June, pp. 683-691.