

# Modeling farmers' response to uncertain rainfall in Burkina Faso: a stochastic programming approach

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## Abstract

Farmers on the Central Plateau of Burkina Faso in West Africa cultivate under precarious conditions. Rainfall variability is extremely high in this area, and accounts for much of the uncertainty surrounding the farmers' decision-making process. Strategies to cope with these risk are typically dynamic. Sequential decision making is one of the most important ways to cope with risk due to uncertain rainfall. In this paper, a stochastic programming model is presented to describe farmers' sequential decisions in reaction to rainfall. The model describes farmers' strategies of production, consumption, selling, purchasing and storage from the start of the growing season until one year after the harvest period. This dynamic model better describes farmers' strategies than static models that are usually applied. This study draws important policy conclusions regarding reorientation of research programs and illustrates how operations research techniques can be usefully applied to study grass root problems in developing countries.

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This paper deals with farmers' strategies on the Central Plateau in Burkina Faso, West Africa. This plateau covers almost a quarter of Burkina Faso's territory and contains almost half of the country's population. The rural people face a gloomy prospect. The prevailing systems of production and distribution do not prevent serious food shortages for the majority of these people, and natural resources have been depleted severely. Despite, or rather owing to this critical situation, farmers have taken important initiatives to try to improve production methods, in particular by making use of local resources. Farmers have incorporated methods of water and soil management, anti-erosion measures and the use of organic manure by integrating livestock and crop cultivation.

Our goal is to establish the extent to which farmers' strategies guarantee enough food for their households and what changes can be made to ensure higher, sustainable levels of food security. We have developed a linear programming model that focuses on one farm household, which is representative of a large number of households on the Central Plateau. This representative (hypothetical) household is here referred to as 'the Household'. The model does not consider 'modern inputs', like chemical fertilizers. The farmers have at their disposal only local resources such as land, labor and manure; almost no capital is invested in agricultural methods. No irrigation is applied. The year has one growing season, which coincides with the rainy season. On the basis of a thorough study of all important village-level studies executed in the past on the Central Plateau (see the references in Section 1), we have constructed a linear programming model (see Maatman et al. (1995, 1996)) for average climatological, environmental and socio-economic conditions. The model corresponds fairly well to actual farmers' strategies with one major exception: the model is of a static nature. All decisions are assumed to be taken at one time, before the start of the growing season. In reality, however, decisions on sowing, resowing, timing of weeding and intensity of weeding are not taken at one time, but progressively during the first weeks of the growing season depending on observed rainfall, germination of the seeds, appearance of weeds, etc. The farmers' production strategies are *dynamic*: decisions are made sequentially. Sequential decision making is one of the most important methods farmers use to control risk due to uncertain rainfall. This paper models this process of sequential decision making. The approach is as follows. Although the farmers' sequential decision making is a continual process in time, three decision-making periods can be distinguished:

Period 1. At the beginning of the growing season during the first rains: Given observed rainfall thus far in this period, what decisions on agricultural production should the farmer make to anticipate uncertain rainfall patterns later in the growing season?

Period 2. Later in the growing season: What decisions should the farmer make given the decisions made in the first period, and given the observed actual rainfall patterns in the second period?

Period 3. The year after the beginning of the harvest, the 'target consumption year': Given the harvest levels, what decisions should be made during this period concerning consumption, storage, selling and purchasing?

The periods are illustrated in Figure 1. Rainfall levels in Periods 1 and 2 are important factors influencing farmers' sequential decisions.

In this paper, we capture the dynamics of this decision-making process by modelling it as a collection of two-stage recourse models. Before describing this approach, Section 1 provides a short survey of the literature on modeling farmers' strategies to cope with risk. Section 2 presents the background and the features of the underlying problem. Section 3 presents the two-stage recourse model. Sections 4 and 5 present results for a static model and the dynamic model, respectively. Section 6 evaluates how much is gained by using our composite model instead of a more simple deterministic model.

Finally, Section 7 presents conclusions and comments on the practical relevance of our results. A complete specification of the model and the numerical results are given in the Appendix.

## 1 Modeling farmers' strategies to cope with risk

Farmers on the Central Plateau face many risks due to factors such as rainfall, plagues of insects, uncertain prices of agricultural produce, uncertain off-farm incomes, etc. The influence of the various risk factors on farmers' strategies differs greatly. Various methods of risk reduction exist, including those aimed at *prevention* of risk (e.g., irrigation), *dispersion* of risk (by diversification of risky activities such as the cultivation of different varieties of crops), *control* of risk (e.g., by sequential decision making) and '*insurance*' against risk. In general, dispersion of risk is only effective if the effects of the different activities are not highly positively correlated (e.g., if a poor rainfall pattern does not have the same effect on yields on all plots). In this study, methods of risk control refer to sequential decisions on (re)sowing and weeding during the growing season, and making use of information that becomes available (e.g. on rainfall, germination of plants, appearance of herbs). Livestock often functions as a method of '*insurance*' against a poor harvest: if a harvest fails, some of the animals can be sold to buy food. In this study, methods of dispersion and risk control are central. Insurance against risk has only been dealt with in terms of stocking cereals at the end of the year.

Before providing a detailed description of our model of the sequential decision-making process of Burkinabe farmers, we first give an overview of the relevant literature on risk analysis, both in theory and applied to Sahel countries. Much has been written on risk in agricultural decision making, and particularly on risk dispersion. Mathematical programming models, which have been shown to be powerful tools for modeling farmers' strategies (see e.g. Jaeger (1984), Adesina and Sanders (1991), Maatman et al. (1995, 1996)), have been adapted to incorporate uncertainty in the model parameters. See Anderson et al. (1977), Hazell and Norton (1986), Boisvert and McCarl (1990), and Hardaker et al. (1991) for an overview of this literature. The most common approaches have involved *expected utility theory* and *risk efficiency methods* (Boisvert and McCarl (1990)). The majority of these 'risk-programming' applications belong to the category of static models: decisions must be made at one time and all (probabilistic) information about the future is assumed to be known.

Control of risk in agriculture, an important issue in this paper, has received much less attention in the literature. Stochastic programming techniques are often used for this kind of risk analysis. Several decision stages are distinguished, with decisions in each stage dependent on available information (previous decisions, observed random events and probabilistic information about the future). Such models, known as recourse models, were introduced by Dantzig in 1955, and have been applied to a variety of problems in areas such as production planning, water management and finance (see the recent text books by Birge and Louveaux (1997), Kall and Wallace (1994) and Prékopa (1995), and the bibliography by Van der Vlerk (1996-2001)). Until recently, the use of multi-period recourse models has been limited owing to computational difficulties. Nowadays, algorithmic progress (see the text books cited above and Wets and Ziemba (1999)) and dramatically increased computing power allow such models to be applied to problems of realistic size.

The literature increasingly recognizes the importance of sequential responses to risk, in particular in the context of farmer's decision making (Hardaker et al. (1991)) and provides many examples in which the recourse approach leads to significantly better results than those obtained with deterministic models in which the random parameters are replaced by their expected values (the so-called expected value problem). We compare the modeling approaches in this study, and find that the recourse model gives superior results, demonstrating the strong influence of uncertainty on decision making (see

Section 6).

Risk in Sahel agriculture is dealt with in several studies. Some of these studies have used statistical (e.g. Chavas et al. (1991)) or econometric methods (e.g. Fafchamps (1993)) to analyze the relationship between planting decisions and yields. The studies have emphasized the sequential nature of decision making, with farmers adapting their decisions to the actual situation. Mathematical programming techniques have been used to build stochastic household models (e.g. Lang et al. (1994), Balcet and Candler (1981)) or village economy models (e.g. Barbier (1994)), but most of these models do not focus on risk control. To our knowledge, few empirical studies have concentrated on sequential decision making in agriculture. In 1968, Cocks discussed an example of an agricultural household in which labor decisions were made sequentially in two stages. He formulated the problem as a discrete, multi-stage, stochastic optimization problem. Rae (1971a, 1971b) further developed this model, and applied it in an analysis of the annual production strategies of a vegetable farm in New Zealand.

Rae has been one of the few researchers to apply discrete, multi-stage, stochastic optimization for agricultural decision making. This approach has not been widely pursued in empirical studies due to the difficulties encountered in adequately capturing the dynamic elements of agricultural decision making. Adesina and Sanders (1991) and Dorward (1994, 1996) incorporated sequential decision making in a risk-programming study. However, the approaches were either very simple, with only a small number of variables, or did not allow for the adaptation of decisions over the course of the season. The model presented in this paper contains multiple decision stages, and in that sense has the same structure as Rae's model. Considerable effort has been made to ensure that the model reflects the farming practices in the region under consideration. Particular attention is paid to the incorporation of the different objectives of subsistence farmers, to accounting for household consumption patterns, and to the role of on-farm consumption of self-produced crops in farmers' strategies. We also model the effect of cropping technologies that allow for different levels of weeding, which is an important risk control method for many farmers in Sahel countries. To our knowledge, this has never before been taken into account in risk programming models.

Our model is an application of well-known techniques from stochastic programming. It is a so-called recourse model with three stages. Such a model is well suited to model the dynamic decision structure of our problem. In the model, no decisions are required before the first observations (i.e. first rainfall) are made. Consequently, the three-stage model can be separated into a collection of two-stage models, each of which corresponds to a particular realization of the first rainfall. As uncertainty is modeled by discrete random variables with only a few possible realizations, no sophisticated tools are needed to solve our models.

The models are normative, i.e. they are instruments to study what the farmers 'should' do. The formulated decision criteria, objectives, constraints, requirements etc. must be realistic and conform farmers' perceptions. A distinction is made between the farmers' actual situation and a potential new situation. The actual situation refers to existing practices as applied by the majority of farmers. In the potential situation, new technologies that are promising but have not yet been adopted in practice by the farmers on a large scale may also be applied. The aim is to develop an instrument that can analyze situations in which a combination of existing and new technologies is applied. A normative model for the actual situation was first developed in order to make certain that the structure of the model and the values of the parameters reflect the factors influencing farmers' decisions in a proper way. The development of this model has involved a repetitive process of interpretation of results, comparison with actual practices and improvement of the models. We have made extensive use of secondary and primary sources in developing our models. In particular, the secondary sources included all village-level studies previously carried out in Burkina Faso: the studies of ICRISAT (e.g. Matlon and Fafchamps (1988); McIntire (1981, 1983); Kristjanson (1987)), ICRISAT and IFPRI (Reardon and

Matlon (1989)); of the programme FSU/SAFGRAD (e.g. Lang, Rotha and Preckel (1984); Magy, Ohm and Sanders (1986); Roth et al. (1986); Roth (1986); Singh et al. (1984); Singh (1988)), of CEDRES of the University of Ouagadougou (Thiobiano, Soulama, Wetta (1988)), of the University of Wisconsin (e.g. Sherman, Shapiro, Gilbert (1987); Delgado (1978); Broekhuysse (1982, 1983); M.J. Dugué (1987); P. Dugué (1989), Kohler (1971), Marchal (1983) Imbs (1986) and Prudencio (1983, 1987)); the results of farming systems research published e.g. in Matlon et al. (1984) and Ohm and Nagy (1985) have been consulted also.

Primary sources were the results of studies and interviews in three villages in the north-west region: Baszaïdo, Kalamtogo and Lankoé, see Ouédraogo et al. (1995). The studies were carried out by the research team zone Nord-Ouest of the Institute of the Environment and Agricultural Research (INERA) of Burkina Faso. The first author of this paper participated in this team for a period of four years. As a result of careful modeling, the outcomes of our model corroborate farmers current practice.

Our model is the first large-scale, multi-period, stochastic programming model to incorporate subsistence farmers' strategies in reaction to rainfall into its structure.

If an important gap is seen to exist between a model and actual practice, this may be due to one of two reasons: either the model is not (yet) correct, or the model is correct but farmers' strategies are not optimal (in the sense described in the model). Since the results of the model presented in this article correspond 'fairly well' with actual practice (see below), two conclusions have been drawn: firstly, the model adequately describes the decision-making system at the farm-level; secondly, there is no reason to suppose that the farmers' strategies are not optimal. This last conclusion complies with the generally accepted view that experienced farmers apply optimal strategies indeed, even if they have to work under very precarious conditions and have very little endowments at their disposal.

The paper deals (only) with the development of a normative model for the analysis of the actual situation. This model for the actual situation has also been extended to include the study of a potential new situation, in which certain new technologies, in particular water and soil management, are adopted. In this extended model, many of the criteria, constraints and conditions are the same as those found in the model for the actual situation, while other elements are different. The justification of the formulation of the influence of these new elements and the estimation of values of newly introduced parameters were based on thorough studies of the new technologies, their impact on yields and labor inputs, costs, etc. (see Maatman et al. 1998). Since the results of our normative model are consistent with the observed strategies of farmers in the actual situation, it seems to be justified to assume that for the new potential situation the carefully adapted model will suggest (near) optimal strategies as well.

The linear programming models (presented in the Appendix) can be solved quickly using standard LP software. They can also be solved in small research centers in developing countries, thus facilitating the application of such models. Indeed, our model is currently being used in this way in several on-going projects to analyze possibilities for agricultural improvement in Burkina Faso (see Maatman et al. (1998)).

## **2 Key elements of the models to describe farmers' strategies on the Central Plateau in Burkina Faso**

The models for the representative household describe crop *production* strategies during the *growing season* and *consumption, storage and marketing* strategies during the *target consumption year* (see Figure 1). Production decisions are modeled by decision variables, one for each combination of the

following characteristics:

- (a) crop choice: possible crops include maize, red sorghum, white sorghum, millet and groundnuts, and the mixed crops red sorghum/cowpeas, white sorghum/cowpeas, millet/cowpeas;
- (b) cultivated land category: specified by the location (low and high lands) and the distance from the compound (less than 100 meters, between 100 and 1000 meters, more than 1000 meters);
- (c) land ownership: common or individual fields;
- (d) applied dose of organic manure: 0, 800, 2000, 4000, 8000 kg per hectare;
- (e) sowing dates: different for each crop and land category;
- (f) levels of intensity of weeding: intensive, or less intensive.

The harvest period consists of three months. An important feature in the models considered is the concept of a *plot*. A plot is a piece of land with the following properties: one of the crops under (a) is grown; it belongs to one of the land categories (b); it is a common or an individual field (c); one of the doses of organic manure (d) is applied; sowing takes place on one of the dates (e). Intensity of weeding (f) is not included in the definition of a plot; it will be handled differently (see Section 3). In this way, a large number of plots can be distinguished. *Representative plots* refer to combinations of crops, land categories, and agricultural methods observed in practice; *alternative plots* refer to other combinations. The area of each plot to be cultivated is a decision variable. Decisions regarding the size of a plot correspond to the production decisions (what, where, how much, how and when cultivation should take place).

Key elements of the models describe the influence of the production factors land, labor and organic manure on the production decisions. The *constraints of land* (per category), of *labor* and of *organic manure* indicate that required amounts of resources cannot exceed available amounts. The growing season is split up into time intervals of two weeks or a month (see Figure 1) in order to formulate the labor constraints for the various time periods. In the labor constraints, the required labor not only includes time spent working on the land, but also the time spent walking to and from the fields. Specific labor constraints are introduced for sowing and land preparation during the first weeks of the growing season, when labor availability is based on available time during days on which the rainfall conditions are favorable for sowing. Organic manure is applied on the communal fields. Fallow practice is another key element of the model: a piece of land is left fallow supplementary to each cultivated plot. The size of this piece of land is assumed to be proportional to the size of the corresponding cultivated plot. Coefficients of proportionality are parameters, with values that depend on category of land, crop and manure level. By selecting parameter values, various scenarios of fallow practice can be analyzed. Here we use parameter values based on observation of actual practice.

Decisions on consumption, storage and marketing are taken during the target consumption year, which is divided into several periods of time. This allows the analysis of the strategies in different periods of the year. Decision variables on consumption correspond to consumption of the various foods produced in each period; marketing decisions reflect the quantities sold or purchased. The nutritive balances express the cereal and the non-cereal consumption in terms of *nutrients* (calories and proteins). In the *stock equations* for all agricultural products, losses as well as seed reserves are included. *Financial balances* also take interest rates into account, as well as non-agricultural incomes and expenses as exogenous parameters.

A few constraints, called *normative* constraints, are included in the models to ensure that calculated patterns of consumption correspond to observed patterns on the Central Plateau. For instance, a

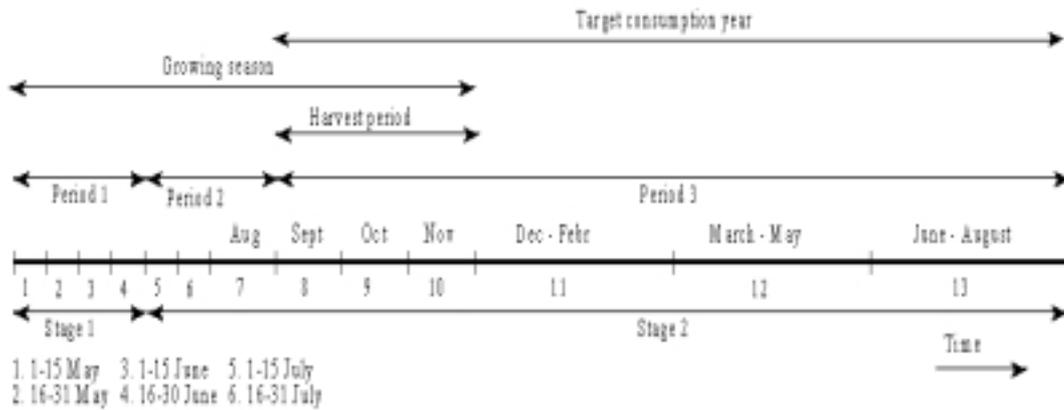


Figure 1: Schematic representation of the planning period and the different decision stages.

restriction is imposed on the consumption of red sorghum, which is mainly used for beer consumption. Another condition requires that part of each meal should consist of cereals. The main objective of all strategies of the Household together is to attain a certain level of self-sufficiency and to try to prevent or, if that is not possible, to minimize shortages of calories and proteins during the target consumption year. If these shortages can be avoided, then a stock is kept for the harvest period of the next year. If these stocks prove sufficient, then the revenues are maximized. In the case of positive revenues, a fraction is spent on keeping a food security safety stock for the next year. All these objectives are dealt with in one objective function and in the formulation of normative constraints.

Data from the sources mentioned in Section 1 have been used to estimate values of parameters in the model. For instance, yields and labor requirements for all plots (i.e. for all crops, categories of land, levels of applied manure, sowing dates, and for levels of intensity of weeding) have been estimated. For alternative plots the values of these parameters have been estimated by extrapolating results of village level studies, and by making use of data from experimental stations. Exogenous selling and purchasing prices refer to producer prices during the harvest period, and to consumer prices during the ‘lean time’ before the next harvest. See Maatman et al. (1995, 1996) for a justification of the estimation of all parameters in the model and their estimated values. The losses due to the traditional grinding of grains play an important role in the analysis. Some grains are hard and thus are difficult to grind unpounded on the millstone. These grains are therefore first pounded and skinned. The losses of nutrients thus incurred are estimated at 25%. It is assumed that the Household can make use of a mill that can grind hard grains in order to avoid such losses.

### 3 Two-stage stochastic models

In practice, farmers make decisions sequentially, depending to a large extent on actual rainfall patterns. Two specific decision moments can be distinguished. The first decisions are made *after observing the dates of the first rains*, but under *uncertainty about rainfall later in the growing season*. Once the latter rainfall is known, a second set of decisions is made. We model this decision process in a number of two-stage stochastic models: for each of a representative set of dates on which first rainfall occurs, a

corresponding two-stage stochastic model is formulated. Consequently, for each of these first rainfall dates, we obtain first-stage decisions that are optimal in a sense that will be clarified below.

The demarcation of the two stages was not evident. If a clear-cut distinction could be made between a first period of sowing and a late period of weeding, then the demarcation would be easy. Late sowing and first weeding may coincide, however, during the growing season. Period 1 has been chosen as the period in which most sowing decisions are taken, and the most important weeding decisions are made in period 2 (see Figure 1).

Observed rainfall in period 1 is called  $r_1$ . The set  $\Omega_1$  contains the possible outcomes of  $r_1$ . The uncertain rainfall in period 2 is considered as a random variable, called  $R_2$ , with realizations  $r_2$  in  $\Omega_2$ . The model for the sequential decision making is a so-called two-stage recourse model; stage 1 refers to period 1, and stage 2 refers to the periods 2 and 3.

In stage 1, which covers the months May and June, production decisions deal with soil preparation, sowing, and early weeding. In practice, these decisions are progressively taken during these two months, and depend in particular on the dates of the first rains. In our model, three situations are distinguished: the growing season starts ‘late’, ‘normal’ or ‘early’, corresponding to three different values of  $r_1$ . The production decisions are different for each of these situations. If the growing season starts late, then less time is available for land preparation and sowing. The number of days that are favorable for sowing is a critical parameter in the labor constraints: the plants will come up if planted during these favorable days and early growth will be successful. Fields may also be sown on other days, but plants will not come through on these fields. These unsuccessful plots may have to be resown later. The number of favorable sowing days depends on  $r_1$ .

The rains in period 2, which covers the months July and August, can be ‘poor’, ‘average’ or ‘good’, corresponding to three possible values of  $R_2$ . The decisions on late sowing and the intensity of weeding are made during this period. It can also be decided to abandon certain plots planted during the first stage. Poor, average or good rainfall, i.e. the value of  $R_2$ , influences the values of various parameters: the time available for late sowing, and in particular the labor for intensive and less intensive weeding. The yield levels (of all plots) also depend on  $R_2$ . The dependence of all these parameters on the three levels of  $R_2$  has been derived from the results of the village level studies referred to in Section 1. Labor requirements for harvesting, which takes place in September, October and November, depend on yield levels. Decisions on consumption, storage and marketing during the target consumption year depend on realized harvest levels, and therefore also depend on rainfall.

As sufficient data exists regarding rainfall on the Central Plateau, more rainfall scenarios, both for  $r_1$  and  $R_2$ , could be distinguished. However, relatively few data are available on the influence of rainfall on yields and labor times, as a function of crop, soil type and agricultural methods. More importantly, the nine scenarios for  $r_1$  and  $R_2$  reflect the division that is generally made by farmers in order to explain the results of the agricultural season (see e.g. Dugué, (1989)). It makes little sense to add more scenarios, since it is difficult to estimate reliable values of the parameters. We note as well that it is not necessarily required to take into account extremely poor rainfall scenarios. Methods of risk insurance anticipate such situations, and these are addressed in other research.

### *Decision variables and model*

The index  $\tau = 1, 2$  refers to period 1 and 2. The plots are defined by their properties (a) – (e) as specified on page 6. We introduce the following sets:

$$J = \{ \text{all plots} \} \tag{1}$$

$$J(\tau) = \{ \text{plots to be sown in period } \tau \}, \quad \tau = 1, 2. \tag{2}$$

Note that  $J(1) \cap J(2) = \emptyset$  and  $J(1) \cup J(2) = J$ . We define for each  $j \in J(1)$  the following first-stage decision variable:

$$SUR1(j) \quad \text{area of plot } j. \quad (3)$$

In the second stage, the decisions on sowing the plots  $j \in J(2)$  and on weeding intensity depend on the observed rainfall  $r_2$  during period 2. For  $j \in J(2)$ , the following second-stage variables are introduced:

$$SUR2(j, r_2) \quad \text{area of plot } j, \text{ if rainfall in period 2 is } r_2, \quad (4)$$

and for  $j \in J$ :

$$\begin{aligned} SURi(j, r_2) & \quad \text{area of that part of plot } j \text{ that will be weeded intensively} \\ & \quad \text{during period 2 if rainfall in period 2 is } r_2; \\ SURE(j, r_2) & \quad \text{area of that part of plot } j \text{ that will be weeded less intensively} \\ & \quad \text{during period 2 if rainfall in period 2 is } r_2. \end{aligned} \quad (5)$$

For the plots that are sown in period 2, it is decided immediately which part will be weeded intensively and which part will be weeded extensively. Hence,

$$SUR2(j, r_2) = SURi(j, r_2) + SURE(j, r_2), \quad j \in J(2). \quad (6)$$

In period 2, it will be decided whether parts of the plots sown in stage 1 will be weeded intensively or extensively, or abandoned. This condition can be written as:

$$SUR1(j) \geq SURi(j, r_2) + SURE(j, r_2), \quad j \in J(1). \quad (7)$$

The inequality in (7) implies that the Household may abandon a part of the plots sown in period 1 (due to a lack of labor if labor requirements for weeding are too high).

The decision variables during period 3 (see Figure 1) correspond to decisions on consumption, sales and purchases of the produce that are taken into account. We define:

$$P = \{\text{maize, red sorghum, white sorghum, millet, groundnuts, cowpeas}\}. \quad (8)$$

As indicated in Figure 1, the time intervals  $t = 1, 2, \dots, 7$  belong to periods 1 and 2. During period 3, the time intervals  $t = 8, 9, \dots, 13$  are distinguished. For  $p \in P$ ,  $t = 8, 9, \dots, 13$ , and rainfall  $r_2 \in \Omega_2$  the following decision variables are introduced:

$$\begin{aligned} CON(p, t, r_2) & \quad \text{consumption of produce } p \text{ during time interval } t, \\ PUR(p, t, r_2) & \quad \text{quantity of produce } p \text{ purchased during time interval } t, \\ SAL(p, t, r_2) & \quad \text{quantity of produce } p \text{ sold during time interval } t. \end{aligned} \quad (9)$$

The definitions of parameters and variables and the formal mathematical model are presented in the Appendix; the equations referred to below appear in Table 1. The constraints of land, including parameters to describe fallow practice, the use of organic manure and labor, both for communal fields and individual fields, are given in (17), (18) and (19) – (22), respectively. Production quantities, corrected for quantities of produce to be reserved as seeds for the next farming season, are defined in (23). Stock equations for each produce and financial balances are formulated in (27) – (29). The constraints (24) – (26) state that farmers, according to practice on the Central Plateau, sell only during the months after harvest and purchase only in the period of the lean time just before the new harvest.

This practice is much in the interest of the traders purchasing from and selling to the farmers on the local market, rather than in the farmers' interest. It often occurs that even in years of shortage farmers have to sell part of the production immediately after the harvest for daily expenses or to repay debts to traders, and then have to buy again later in the year when prices are much higher. This phenomenon is well known on the Central Plateau and in many other regions of Africa (see e.g. Yonli (1997)).

The objective of our model is to minimize (expected) deficits of various nutrients during the planning period including the harvest period of the next farming season. The constraints (30) to (36) model nutritive and consumption requirements, which are formulated in terms of three different measures of possible deficits:

- (i) deficits of nutrients in each period of the target consumption year,
- (ii) deficits during the harvest period of the next farming season,
- (iii) deficit of *auto-subsistence cereal production*, which is defined as the minimal quantity of staple cereals to be produced by the Household itself.

Possible deficits are modelled as second-stage or *recourse* variables with corresponding recourse costs, because they depend on rainfall  $R_2$  in the second period. In addition, the objective function also contains a term for the (expected) net revenues during the target consumption year, which are to be maximized. If the Household has positive revenues, these will partly be spent on the construction of a safety stock (see (37)).

Although it may be possible to specify unit costs for deficits of type (i), (ii) and (iii) (the fourth term in (16) is already in monetary units), we have chosen the coefficients in the objective function in such a way that highest priority is given to the minimization of shortages in the target consumption year, and thereafter in decreasing order to (ii), (iii), and net revenues. Shortages  $DEF(n, t, r_2)$  are first minimized. If these shortages are zero, the household switches to minimizing the shortages in the harvest period of the next year. The coefficients in the objective function (16) are chosen to reflect this ordering.

The two-stage stochastic models of Table 1 can be summarized as follows. The following vectors are introduced:

- $x_1$  first-stage variables, corresponding to decisions taken in stage 1;
- $x_2$  second-stage variables, corresponding to decisions taken in stage 2.

For each  $r_1 \in \Omega_1$  values of  $x_1$  are computed that solve

$$\min_{x_1} \{Ez(x_1, R_2) \mid A_1(r_1)x_1 = b_1(r_1), x_1 \geq 0\} \quad (10)$$

where  $E$  refers to the expectation with regard to  $R_2$  and  $z$  is the value function of the second-stage problem. For any realization  $r_2 \in \Omega_2$  and first-stage decision  $x_1$ ,

$$z(x_1, r_2) = \min_{x_2} \{c^\top x_2 \mid B_2(r_2)x_2 = b_2(r_2) - B_1(r_2)x_1, x_2 \geq 0\}. \quad (11)$$

$A_1(r_1)$ ,  $B_1(r_2)$  and  $B_2(r_2)$  are matrices,  $b_1(r_1)$ ,  $b_2(r_2)$  and  $c$  are vectors with elements that depend on  $r_1$  and  $r_2$ . Their contents will be specified below. Thus our model leads to a first-stage decision that has minimal expected second-stage costs (there are no first-stage costs).

We recall that rainfall in period 2 can be 'poor', 'average', or 'good'. This is modelled by the discrete random variable  $R_2$ , which has this three possible realizations, denoted by  $r_2 \in \Omega_2$ . The

discretization is chosen in such a way that all outcomes have equal probability, i.e.,  $f(r_2) := \Pr(R_2 = r_2) = 1/3$  for all  $r_2 \in \Omega_2$ .

Since  $R_2$  is a discrete random variable, it follows that for each  $r_1 \in \Omega_1$ , the recourse problem (10)-(11) is equivalent to a deterministic large-scale linear programming problem of the following form:

$$\min_{x_1, x_2(r_2)} \left\{ \sum_{r_2 \in \Omega_2} f(r_2) c^\top x_2(r_2) \mid \begin{array}{l} A_1(r_1)x_1 = b_1(r_1), \\ B_2(r_2)x_2(r_2) = b_2(r_2) - B_1(r_2)x_1, \quad r_2 \in \Omega_2, \\ x_1 \geq 0, \quad x_2(r_2) \geq 0, \quad r_2 \in \Omega_2 \end{array} \right\}. \quad (12)$$

This model is completely specified in Table 1. In (12),  $x_1$  represents the first-stage variables *SUR1* defined in (3), whereas  $x_2$  are the second-stage variables in the model. (The vectors  $x_1$  and  $x_2$  also contain the appropriate slack variables.) The constraints  $A_1(r_1)x_1 = b_1(r_1)$  correspond to the first-stage constraint (19), and  $B_2(r_2)x_2(r_2) = b_2(r_2) - B_1(r_2)x_1$  for all  $r_2 \in \Omega_2$  to the other constraints. The vector  $c$  contains the weighting coefficients in (16).

The two-stage model has 2,724 variables and 1,252 constraints. The linear programming problems were formulated in GAMS and solved with MINOS5 (Brooke et al. (1992)). In Table 2, some computational results are presented, which we discuss in Sections 4 and 5. A comparison is made between the results of the two-stage stochastic models and static models. In a static model,  $\bar{r}_1$  and  $\bar{r}_2$  are assumed to be known, hence the static model can be written as:

$$\min_{x_1, x_2} \left\{ c^\top x_2 \mid A_1(r_1)x_1 = b_1(r_1), B_2(\bar{r}_2)x_2 = b_2(\bar{r}_2) - B_1(\bar{r}_2)x_1, x_1 \geq 0, x_2 \geq 0 \right\}. \quad (13)$$

The model (13) is called the *average static model* for average rainfall  $\bar{r}_1$  and  $\bar{r}_2$  ( $\bar{r}_1 = \text{normal}$ ,  $\bar{r}_2 = \text{average}$ ).

The values of all parameters in (12) and (13) and the justification of their estimates are given in Maatman et al. (1995, 1996). It is recalled that the estimation has been based on a thorough exploration of all important village level studies and other secondary sources, and complemented by the use of primary field data (see Section 1). Crop yields and labor inputs for various levels of rainfall and weeding are key parameters. Estimates of crop yields reflect various crop characteristics. Millet, for example, is more drought resistant than sorghum and is more tolerant of weeds.

The computation times (on a Pentium 200 Mhz with 64MB internal memory) were 90 seconds (10,000 iterations) for the two-stage models, and 10 seconds (1000 iterations) for the static models. Results of both static and two-stage models are discussed in the following sections.

## 4 Results of static models

The presentation of the results in Sections 4 and 5 consists of a discussion of the main characteristics of the farmers' strategies. The policy implications of the results will be discussed at the end of Section 5.

The results of the average static model show that the Household can just manage to avoid shortages of nutrients (calories and proteins) during the target consumption year in a year of average rainfall. Production alone is not enough: all revenues from other sources are used to buy cereals during the 'lean time'. No reserve stocks can be kept.

A remarkable outcome is the *heterogeneity of agricultural strategies*, i.e. the cultivation of different crops, both sole-cropped and intercropped, on different soil types, and using a great diversity of growing methods (sowing periods, quantities of organic manure, intensive and less intensive weeding). The great diversity in agricultural activities, in response to a complex range of objectives and

constraints, is a key element of the farmers' actual strategies on the Central Plateau. Another result that conforms to observations made in practice is the need to buy cereals later in the year.

Notwithstanding the heterogeneity of the cropping strategies of the Household, some general tendencies can be observed. Millet and white sorghum are cultivated on the highlands with no, or only very low levels of organic manure, red and white sorghum with moderate fertilization on the low lands and maize on some small plots at a short distance from the household with high doses of organic manure. Cowpeas are cultivated as an intercrop on both millet and sorghum fields. Maize is an important crop, because it is harvested during the first weeks of the harvest period just before the harvest of the (large) millet fields. The cultivation of *early crops* like maize is urgently required because no stocks remain from the previous year.

We have also computed results for static models with other scenarios of  $r_1$  and  $r_2$ . The most conspicuous result is the difference between the results of 'average' and 'poor' rainfall scenarios in period 2. For the average rainfall scenario a large part of production consists of white sorghum. For the poor rainfall scenario, however, almost all white sorghum is replaced by millet. This effect is explained in the next section.

## 5 Results of the two-stage stochastic models

The results of the two-stage model for average rainfall in the first period differ from those of the average static model in two important aspects:

- the increased importance of millet cultivation;
- the extension of the area sown in the first period.

The predominance of millet, both sole-cropped or intercropped with cowpeas, may be explained by its resistance to drought stress and its tolerance to weeds, which is important when rainfall in the second period is average or good (see below). Hence, millet has properties that are favorable in all possible rainfall scenarios and thus its cultivation mitigates risk.

With regard to crop choice, the results to a great extent resemble those of the 'poor' rainfall scenario discussed above. The resemblance with the pessimistic scenario can be well understood. The choice of crops is mainly determined so as to minimize deficits, as reflected by the weights in the objective function. Much can be gained by a good crop choice in poor rainfall years when deficits are large. On the other hand, if rainfall is average or good, deficits are not very sensitive to crop choice: they will be relatively small or even zero (see Table 2). Since each rainfall scenario is equally likely, minimization of expected deficits comes down to minimization of deficits in poor rainfall years.

For average rainfall in period 1, when  $r_1 = \bar{r}_1$ , the outcome of the two-stage stochastic model is that 6% more land is sown during period 1 than in the average static model (see Table 2, for  $r_1 = \text{normal}$ ). Sensitivity analysis shows that the area sown in period 1 for a normal start of the rainfall season is very much restricted by the land constraints. If more land was available, the Household would extend the area sown in the first period even further. Again, sowing a large area in the first phase optimizes production levels when rainfall in period 2 is poor. In that case, poor rainfall conditions limit the growth of weeds and all weeding can be done intensively. However, when rains in period 2 are average or good, labor requirements for weeding become a problem. In that case, the Household is forced to start weeding some fields later, and to weed less carefully. Some fields cannot be weeded in time when rains in period 2 are good and these fields must be abandoned.

In the previous section it was mentioned that diversification of cropping systems is characteristic for the farmers' strategies on the Central Plateau. The scope of further diversification to reduce risk is

limited. The cultivation of millet and of large areas are the most important strategies to minimize risk. Both strategies make the Household less sensitive to the effects of rainfall, in particular poor rainfall, in period 2.

The production strategies differ considerably according to rainfall patterns, see Table 2. For the different scenarios of  $r_1$  and  $r_2$  this variability can best be illustrated for two extreme situations:

1. A late start of the growing season and poor rainfall in period 2: time for sowing is limited. Poor rainfall conditions limit the growth of weeds, all weeding can be done intensively. Yields and production are low.
2. An early start of the growing season and good rainfall in period 2: much more labor time is available for sowing. Labor time for weeding is restricted. In fact, the Household is obliged to abandon part of its fields and another part is weeded less intensively. Yields and production are relatively high.

These results show that farmers indeed should react significantly to rainfall patterns: their strategies should be flexible. The correspondence of the results described in Chapter 5 to field practices can be seen from various reports dealing with farmers' strategies in the region concerned as well as from observation of farmers. The results have been discussed at length with farmers and other experts in the field. Crop areas and labor profiles, for example, closely correspond to those observed in various field studies (e.g. Matlon and Fafchamps (1988), Singh (1988), INERA/RSP/Nord-Ouest (1997), see also the agricultural surveys executed by MARA/DSAP (1991–1996). The extensive and flexible character of cropping strategies, as described by Marchal (1983, 1989) and Dugué (1989), is very well reflected in the outcomes of our model. Strategies of diversification and risk control are also emphasized in the studies by Lang et al. (1984), Kristjanson (1987), and Fafchamps (1989)). The more realistic nature of the two-stage recourse models in comparison to the static models is to a large extent due to the more precise formulation of the labor constraints in period 1 and 2. It may be concluded that the two-stage stochastic model is more convincing than the static model with average parameter values, because the static model does not lead to the preferred strategies.

The results discussed above have some important policy implications:

- The 'extensification' (as opposed to intensification; extension of land combined with non-intensive ways of cultivation) of agriculture is an effective means to control risk due to uncertain rainfall. This partly explains why this phenomenon is so widespread in the region and why farmers may be reluctant to shift from 'extensification' to intensification of agricultural practice.
- Heterogeneity and flexibility are key elements of the farmers' strategies in coping with uncertain rainfall regimes and fragile environmental conditions. In development projects these elements have to be safeguarded.
- Sensitivity analysis showed that constraints on land, labor and organic manure limit the potential of the existing farming systems to stop degradation of natural resources and to maintain or increase agricultural productivity. Optimal use of organic manure from livestock is of high priority. Nevertheless, external inputs, like chemical fertilizers, will also be needed if agricultural production is to be enhanced.

## **6 The value of using multiple recourse models**

In this section we evaluate how much is gained by using our composite modeling approach, characterized by

- (i) separate models for each realization of  $r_1$ ,
- (ii) for each realization of  $r_1$ , a two-stage recourse model that captures the uncertainty with respect to  $R_2$ ,

instead of one deterministic model (the average static model). To this end, we utilize several concepts from literature. As mentioned, the main goal of our models is to suggest strategies that are optimal in practice. As discussed in the previous section, the recourse models indeed outperform the static models in this respect. In addition, the following quantitative measures indicate that the recourse models are preferable in this case study.

First of all, for each realization of  $r_1$ , we find that the optimal value of the static model with average rainfall in period 2 is lower (i.e., better) than that of the corresponding recourse model. This is not surprising, since the optimal value of a static model is based on the false assumption that the future rainfall pattern is known. Assuming that the recourse model is an adequate representation of the decision problem at hand, the ‘true’ objective value of a solution of such a static model can be determined as follows.

For each fixed  $r_1 \in \Omega_1$ , consider the static model with rainfall in period 1 equal to  $r_1$  and rainfall in period 2 equal to  $\bar{r}_2$  (for  $r_1 = \bar{r}_1$  this is the average static model (13)). Let  $\bar{x}_1(r_1)$  denote an optimal first-stage solution of this model. Then the true objective value corresponding to  $\bar{x}_1(r_1)$  is obtained by substituting  $\bar{x}_1(r_1)$  into the two-stage model: it is given by

$$\min_{x_2(r_2)} \left\{ \sum_{r_2 \in \Omega_2} f(r_2) c^\top x_2(r_2) \mid \begin{array}{l} B_2(r_2) x_2(r_2) = b_2(r_2) - B_1(r_2) \bar{x}_1(r_1), \quad r_2 \in \Omega_2, \\ x_2(r_2) \geq 0, \quad r_2 \in \Omega_2. \end{array} \right. \quad (14)$$

The difference between (14) and the optimal value of the corresponding two-stage model (12) is known as the *value of the stochastic solution* (VSS), which is always non-negative (Birge and Louveaux (1997)). This can be understood from the fact that a solution of a two-stage model anticipates the uncertainty in the future, whereas  $\bar{x}_1(r_1)$  does not take into account the uncertainty of future rainfall.

For our problem (with  $r_1 = \bar{r}_1$ ), the VSS equals 17% of the optimal value of the objective function. For the models with early and late first rains the VSS is 25% and 3%, respectively. These values indicate that it is indeed useful to use stochastic models for this problem. The VSS increases considerably when rainfall in the first stage becomes more favorable, thus illustrating the effect of first-stage rainfall on farmers’ opportunities to anticipate poor rainfall in the second stage. Of course, when rainfall is late, the possibility to apply risk-controlling strategies (sowing of large areas) becomes limited.

Numerical experiments also corroborate our decision to use separate models for each realization of  $r_1$ . Indeed, if the first-stage optimal solution  $\bar{x}_1$  of the average static model is substituted in the two-stage model with  $r_1 = \text{late}$ , this model becomes infeasible due to the labor constraints for the first period. This is a strong qualitative indication that our approach is preferable.

Finally, we use the concept of the *expected value of perfect information* (EVPI) to investigate how important the role of randomness is in our model. EVPI can be interpreted as the price one would be willing to pay for complete information on future events if this were possible. The (expected) value of perfect information has been treated in the literature in different contexts (see e.g. Kristjanson (1987), Chavas and Pope (1984), Chavas et al. (1991)). The concept of EVPI as considered here is commonly used in stochastic programming (Kall and Wallace (1994), Prékopa (1995), Birge and Louveaux (1997)).

In order to compute the EVPI (with respect to  $R_2$ ) for each of the two-stage models, we first determine solutions of all models under perfect information. That is, for each fixed  $r_1 \in \Omega_1$ , and each

possible realization  $r_2 \in \Omega_2$ , we solve the deterministic problem

$$v(r_1, r_2) = \min\{c^\top x_2(r_1, r_2) \mid \begin{aligned} A_1(r_1) x_1(r_1, r_2) &= b_1(r_1), \\ B_2(r_2) x_2(r_1, r_2) &= b_2(r_2) - B_1(r_2) x_1(r_1, r_2), \\ x_1(r_1, r_2), x_2(r_1, r_2) &\geq 0 \end{aligned}\}. \quad (15)$$

Next, we calculate the expected optimal values  $Ev(r_1, R_2) = \sum_{r_2 \in \Omega_2} f(r_2)v(r_1, r_2)$ ,  $r_1 \in \Omega_1$ . For each  $r_1 \in \Omega_1$ , the EVPI with respect to  $R_2$  is given by the difference between the optimal value of the corresponding recourse model and the expected optimal value  $Ev(r_1, R_2)$ . It is easy to see that the EVPI is non-negative.

Computations for the various values of  $r_1$  show (see Table 3), that the EVPI equals about 7–9% of the corresponding optimal value. When interpreting these results, we should bear in mind that we are comparing (expected) objective values representing multiple objectives weighted by more or less arbitrary weights. On the other hand, the EVPI refers to food shortages. In a precarious situation 7–9% less food shortage can be of crucial importance.

## 7 Concluding remarks

We would like to comment briefly on the scientific interest and the practical usefulness of our approach, and of the application of multi-stage stochastic models in particular.

We will first discuss the scientific interest of our research. Multi-stage recourse models have been applied successfully to various problems, but rarely to socio-economic problems. The scientific challenge of the present study has been to explore whether such models can be usefully applied to real complex socio-economic problems, such as rain-fed subsistence farming in a Sahel country.

When we started our research, we had reservations about applying stochastic programming models to study the sequential decisions of farmers: the processes might be too complex to be modeled, it might not be possible to estimate parameters on yields and labor inputs that depend on rainfall, etc. Would it be possible to develop a model that would correspond to the actual social, economic and environmental farming conditions? Our initial reservations could be discarded due to the fact that the sequential decision making by the farmers is such a vital element in the way farmers in Burkina Faso cope with risk. That it has been possible to set up a stochastic programming model that can describe the complex sequential decision-making process fairly well has been quite a revealing result in itself. It appeared to be possible to limit the size of a number of stages in the stochastic programming model, thus allowing the model to be handled and well understood. The necessary data was available. Observed practices could be apprehended. We conclude that stochastic programming models can also be usefully applied to analyze complex socio-economic problems. For agricultural scientists it is an important result that, for the Sahel rainfall conditions, the recourse models describe farmers' strategies better than deterministic models.

The discussion of the practical usefulness of our research deals with two issues: the applicability of the model and how the results of the study have and will be used. The main target of the present study was the development of a proper instrument of analysis: a model that corresponds satisfactorily to farmers' strategies in practice. Therefore, the analysis discussed in this paper was for the most part restricted to *existing* agricultural practices. Once such a model has been constructed and verified, it can be used as a tool to study new strategies. Because the food situation on the Central Plateau is extremely delicate, there is an urgent need to improve agricultural methods. The two-stage stochastic models described in this paper have also been used to study alternative methods such as anti-erosion methods to regain land (Maatman et al. (1998)).

Researchers applying operations research techniques often claim that they offer a tool of analysis to ‘decision makers’ who can use it to develop and implement new strategies. In our case study of Burkina Faso, the decision makers are found at many levels: the farmers themselves in cooperatives, non-governmental organizations, agricultural research stations, provincial and national extension services and others. Implementation of changes in farmers’ production systems is a long-term process in which all these decision makers may be involved. No single operations research model, nor its conclusions, will immediately alter these processes. Neither will it directly influence the decisions of regional or national policy makers. However, the study described in this paper has undeniably influenced research and development programs in Burkina Faso, and some of the conclusions continue to be used in policy debates. Our modeling research was integrated in the national program for agricultural research in Burkina Faso, as carried out by the National Institute of Agricultural Research (INERA) and the University of Ouagadougou. Cooperation with researchers at these institutes has not only resulted in a permanent dialogue about the stepwise improvement of the model, but also about reorientation of policies and field research carried out by INERA.

The research focused on understanding the rationale of the farmers’ strategies and on policy implications. Within development and agricultural institutions in Burkina Faso there is a growing consensus about the policy implications discussed at the end of Section 5. This is apparent, among others from the reorientation of the national agricultural research program. For example, the integration of cattle farming and crop cultivation is now a major subject of field research, and on-farm experiments are directed at adapting technologies to the specific circumstances of individual farmers. The results of this study have contributed to this reorientation and show in a comprehensive way why this orientation is justified.

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## Appendix

### Definition of the variables and parameters of the model in Table 1.

The index  $\tau$  corresponds to period 1 and 2,  $t = 1, 2, \dots, 13$  to the time intervals indicated in Figure 1,  $s$  to the categories of land,  $l$  to the mode of ownership,  $p$  to the produce introduced in (8),  $n$  to the nutrients, and  $j$  to the plots in (1) and (2). We define:

#### Sets:

$S$	= {land categories}
$L$	= {1,2}; 1 = common fields, 2 = individual fields
$J$	= {all plots}
$J(\tau)$	= {plots to be sown in period $\tau$ }, $\tau = 1, 2$
$JS(s, \tau)$	= plots of land category $s \in S$ , sown in period $\tau$ , $\tau = 1, 2$
$JL(l, \tau)$	= plots with mode of ownership $l \in L$ , sown in period $\tau$ , $\tau = 1, 2$
$P$	= {maize, red sorghum, white sorghum, millet, groundnuts, cowpeas}
$P_{cer}$	= {maize, red sorghum, white sorghum, millet}
$N$	= {kilocalories, proteins}

#### Variables:

For  $p \in P, t = 8, 9, \dots, 13, n \in N$  and  $r_2 \in \Omega_2$ , the following variables are defined:

$SUR1(j)$	area of plot $j$ (ha), $j \in J(1)$ ;
$SUR2(j, r_2)$	area of plot $j$ (ha), $j \in J(2)$ ;
$SURi(j, r_2)$	area of that part of plot $j$ (ha) that will be weeded intensively during period 2, $j \in J$ ;
$SURe(j, r_2)$	area of that part of plot $j$ (ha) that will be weeded less intensively; during period 2, $j \in J$ ;
$CON(p, t, r_2)$	consumption of produce $p$ (kg) during time interval $t$ ;
$PUR(p, t, r_2)$	quantity of produce $p$ (kg) purchased during time interval $t$ ;
$SAL(p, t, r_2)$	quantity of produce $p$ (kg) sold during time interval $t$ ;
$PRO(p, t, r_2)$	harvest of product $p$ (in kg) in time interval $t$ , available for consumption or sale, $t = 8, 9, 10$ ;
$ST(p, t, r_2)$	stock of product $p$ at the end of time interval $t$ (in kg);
$STR(p, r_2)$	volume of the stock of product $p$ (in kg) saved at the end of time interval 13 to contribute to food needs in the harvest period of next farming season;
$SAFST(p, r_2)$	volume of the safety stock of product $p$ (in kg) reserved at the end of

time interval 13 to meet food requirements after the harvest period of the next farming season;

$FIN(t, r_2)$	financial resources of the Household at the end of time interval $t$ (in francs CFA);
$REV(r_2)$	net revenues during the target consumption year (in francs CFA);
$DEF(n, t, r_2)$	deficit of nutrient $n$ in time interval $t$ (in proteins or kilocalories);
$DEFPR(n, r_2)$	deficit of nutrient $n$ during the harvest period of the next farming season, if the consumption of the Household was based only on the agricultural stocks at the end of time interval 13 (in proteins or kilocalories);
$DEFPR(r_2)$	deficit of auto-subsistence production (in kg);

### Parameters:

For  $j \in J, s \in S, l \in L, t = 1, 2, \dots, 13, n \in N, r_2 \in \Omega_2$ :

$av(s)$	available area of soil type $s$ in hectare (ha);
$\lambda(j)$	ratio of the area of the fallow supplement of plot $j$ to the surface area of the cultivated plot $j \in J$ ;
$avman$	quantity of manure available to the farm in kg;
$man(j)$	quantity in kg/ha of manure applied on plot $j, j \in J$ ;
$avlab(l, t)$	available labor during time interval $t$ for farming activities (in hours) on common fields ( $l = 1$ ) or individual fields ( $l = 2$ ), $t = 1, \dots, 10$ ;
$lab(j, t)$	required labor in time interval $t$ to cultivate 1 ha of plot $j$ , $t = 1, \dots, 4$ (hrs/ha);
$labi(j, t, r_2)$	labor required to cultivate 1 ha of plot $j$ intensively in time interval $t$ , $t = 5, \dots, 10$ (hrs/ha);
$labe(j, t, r_2)$	labor required to cultivate 1 ha of plot $j$ less intensively in time interval $t$ , $t = 5, \dots, 10$ (hrs/ha);
$labsow(j, t)$	labor required in time interval $t, t = 1, \dots, 5$ , for preparing and sowing 1 ha of plot $j$ (hrs/ha);
$sowday(t, r_1)$	number of favorable days in time interval $t$ for preparing and sowing the fields if rainfall in period 1 equals $r_1$ , for $t = 1, \dots, 4$ ;
$sowdays(t, r_2)$	number of favorable days in time interval $t$ for preparing and sowing the fields if rainfall in period 2 equals $r_2$ , for $t = 5$ ;
$dur(t)$	duration (in number of days) of time interval $t$ ;
$ylidi(j, p, t, r_2)$	quantity of product $p$ harvested in time interval $t$ on 1 ha of plot $j$ if it is weeded intensively, if rainfall in period 2 equals $r_2, t = 8, 9, 10$ (kg/ha);
$ylde(j, p, t, r_2)$	quantity of product $p$ harvested in time interval $t$ on 1 ha of plot $j$ if it is weeded extensively, if rainfall in period 2 equals $r_2, t = 8, 9, 10$ (kg/ha);
$\gamma(j, p, t)$	quantity of product $p$ to be reserved per ha of plot $j$ in time interval $t$ ,

	$t = 8, 9, 10$ (kg/ha);
$f(p, t)$	fraction of the stock of product $p$ lost in time interval $t$ due to storage losses, $t = 8, \dots, 13$ ;
$\rho(t)$	interest rate in time interval $t$ on the capital deposited, $t = 8, \dots, 13$ ;
$nci(t)$	non-cropping incomes received by the Household at the end of time interval $t$ , $t = 8, \dots, 13$ (in francs CFA);
$nfe(t)$	non-food expenses of the Household during time interval $t$ , $t = 8, \dots, 13$ , (in francs CFA);
$prs(p, r_2)$	selling price that the farm expects to receive when selling 1 kg of product $p$ , if rainfall in period 2 equals $r_2$ (in francs CFA/kg);
$prp(p, r_2)$	purchasing price that the farm expects to pay for 1 kg of product $p$ , if rainfall in period 2 equals $r_2$ (in francs CFA/kg);
$fn(7)$	financial resources of the Household at the end of time interval 7 (in francs CFA);
$val(p, n)$	contents of nutrient $n$ per consumed kg of product $p$ (in kilocalories or proteins per kg);
$dem(n, t)$	demand of nutrient $n$ by the Household during time interval $t$ , $t = 8, \dots, 13$ (in kilocalories or proteins);
$demr(n)$	demand of nutrient $n$ by the Household during the harvesting period of the next growing season (in kilocalories or proteins);
$\theta_1(n)$	fraction of the demand of nutrient $n$ to be satisfied by consuming products $p \in P$ ;
$\theta_2(n)$	fraction of the consumption of nutrient $n$ taken by the consumption of the staple cereals, white sorghum, millet and maize;
$\theta_3(n)$	critical minimum fraction of the consumption of nutrient $n$ ;
$\alpha$	fraction of the consumption of staple cereals (white sorghum, millet and maize) to be produced by the Household itself;
$\beta$	fraction of the revenues to be used to build up a safety stock;
$maxrs(t)$	maximum red sorghum quantity which can be consumed in time interval $t$ (in kg);
$w(n), w_1(n), w_2, w$	weighting coefficients in the objective function.

Parameter values are taken from Maatman et al. (1995, 1996).

**Table 1: Stochastic linear programming models for sequential decision making by the Household in Burkina Faso**

For each  $r_1 \in \Omega_1$ , minimize:

$$\sum_{r_2 \in \Omega_2} \frac{1}{3} \left( \sum_{n \in N} \sum_{t=8, \dots, 13} w(n) \cdot DEF(n, t, r_2) + \sum_{n \in N} w_1(n) \cdot DEFR(n, r_2) + w_2 \cdot DEFPR(r_2) - w \cdot REV(r_2) \right) \quad (16)$$

while, for all  $r_2 \in \Omega_2$ ,  $p \in P$ , and  $n \in N$ :

$$\sum_{j \in JS(s,1)} (1 + \lambda(j)) \cdot SUR1(j) + \sum_{j \in JS(s,2)} (1 + \lambda(j)) \cdot SUR2(j, r_2) \leq av(s), \quad s \in S \quad (17)$$

$$SUR2(j, r_2) = SURi(j, r_2) + SURe(j, r_2), \quad j \in J(2)$$

$$SUR1(j) \geq SURi(j, r_2) + SURe(j, r_2), \quad j \in J(1)$$

$$\sum_{j \in J(1)} man(j) \cdot SUR1(j) + \sum_{j \in J(2)} man(j) \cdot SUR2(j, r_2) \leq avman \quad (18)$$

$$\sum_{j \in JL(l,1)} lab(j, t) \cdot SUR1(j) \leq avlab(l, t), \quad l = 1, 2, t = 1, \dots, 4 \quad (19)$$

$$\sum_{j \in JL(l,1) \cup JL(l,2)} \left( labi(j, t, r_2) \cdot SURi(j, r_2) + labe(j, t, r_2) \cdot SURe(j, r_2) \right) \leq avlab(l, t), \quad l = 1, 2, t = 5, \dots, 10 \quad (20)$$

$$\sum_{j \in JL(l,1)} labsow(j, t) \cdot SUR1(j) \leq sowday(t, r_1) \cdot avlab(l, t) / dur(t), \quad l = 1, 2, t = 1, \dots, 4 \quad (21)$$

$$\sum_{j \in JL(l,2)} labsow(j, t) \cdot SUR2(j, r_2) \leq sowdays(t, r_2) \cdot avlab(l, t) / dur(t), \quad l = 1, 2, t = 5 \quad (22)$$

$$PRO(p, t, r_2) = \sum_{j \in J} \left( yldi(j, p, t, r_2) - \gamma(j, p, t) \right) \cdot SURi(j, r_2) + \left( ylde(j, p, t, r_2) - \gamma(j, p, t) \right) \cdot SURe(j, r_2), \quad t = 8, 9, 10 \quad (23)$$

$$PRO(p, t, r_2) = 0, \quad t = 11, 12, 13 \quad (24)$$

$$PUR(p, t, r_2) = 0, \quad t = 9, 10, 11 \quad (25)$$

$$SAL(p, t, r_2) = 0, \quad t = 8, 9, 10, 12, 13 \quad (26)$$

$$ST(p, t, r_2) = (1 - f(p, t)) \cdot ST(p, t - 1, r_2) + (1 - f(p, t)/2) \\ \times \left( PRO(p, t, r_2) + PUR(p, t, r_2) - SAL(p, t, r_2) - CON(p, t, r_2) \right), \quad t = 8, \dots, 13 \quad (27)$$

$$ST(p, 13, r_2) = STR(p, r_2) + SAFST(p, r_2) \quad (28)$$

$$FIN(t, r_2) = (1 + \rho(t)) \cdot FIN(t - 1, r_2) + (1 + \rho(t)/2) \\ \times \left( \sum_{p \in P} \left( prs(p, r_2) \cdot SAL(p, t, r_2) - prp(p, r_2) \cdot PUR(p, t, r_2) \right) + nci(t) - nfe(t) \right), \\ t = 8, \dots, 13 \quad (29)$$

$$REV(r_2) = FIN(13, r_2) - fin(7)$$

$$DEF(n, t, r_2) \geq \theta_1(n) \cdot dem(n, t) - \sum_{p \in P} CON(p, t, r_2) \cdot val(p, n), \quad t = 8, \dots, 13 \quad (30)$$

$$DEFR(n, r_2) \geq \theta_1(n) \cdot demr(n) - \sum_{p \in P} STR(p, r_2) \cdot val(p, n) \quad (31)$$

$$CON(RS, t, r_2) \leq maxrs(t), \quad t = 8, \dots, 13 \quad (32)$$

$$ST(RS, 13, r_2) \leq maxrs(8) + maxrs(9) + maxrs(10) \quad (33)$$

$$\sum_{p \in P_{cer} \setminus \{RS\}} CON(p, t, r_2) \cdot val(p, n) \geq \theta_2(n) \cdot \sum_{p \in P \setminus \{RS\}} CON(p, t, r_2) \cdot val(p, n), \\ t = 8, \dots, 13 \quad (34)$$

$$\sum_{p \in P} CON(p, t, r_2) \cdot val(p, n) \geq \theta_3(n) \cdot dem(n, t), \quad t = 8, \dots, 13 \quad (35)$$

$$DEFPR(r_2) \geq \alpha \cdot \sum_{p \in P_{cer} \setminus \{RS\}} \sum_{t=8}^{13} CON(p, t, r_2) - \sum_{p \in P_{cer} \setminus \{RS\}} \sum_{t=8}^{10} PRO(p, t, r_2) \quad (36)$$

$$\sum_{p \in P_{cer}} prp(p, r_2) \cdot SAFST(p, r_2) \geq \beta \cdot REV(r_2) \quad (37)$$

$$SUR1(j) \geq 0, \quad j \in J(1)$$

$$SUR2(j, r_2) \geq 0, \quad j \in J(2)$$

$$SURi(j, r_2), \quad SURe(j, r_2) \geq 0, \quad j \in J$$

$$CON(p, t, r_2), \quad SAL(p, t, r_2), \quad PUR(p, t, r_2), \quad FIN(t, r_2), \quad ST(p, t, r_2) \geq 0, \quad t = 8, \dots, 13$$

$$STR(p, r_2), \quad SAFST(p, r_2), \quad DEF(n, t, r_2), \quad DEFR(n, r_2), \quad DEFPR(r_2) \geq 0, \quad t = 8, \dots, 13$$

**Table 2: Some results of the two-stage models**

	Results average static model		Results of two-stage models (see (12))							
	(normal)	late	normal		early					
<i>Rainfall period 1</i>										
Value of the objective function <sup>1</sup> ( $\times 10^5$ )	2.74	12.16	8.77		7.94					
Cultivated area <sup>2</sup> (ha)	2.86	2.34	3.03		3.00					
- Red Sorghum	0.19	0.18	0.18		0.11					
- White Sorghum	0.77	0.30	0.11		-					
- White Sorghum/cowpea	0.62	-	0.02		0.24					
- Millet	-	1.05	0.19		0.19					
- Millet/cowpea	1.28	0.81	2.53		2.46					
<i>Rainfall period 2</i>	(aver.)	bad	aver.	good	bad	aver.	good	bad	aver.	good
Cultivated area <sup>3</sup> (ha)										
- Maize	0.13	0.13	0.16	0.10	0.13	0.15	0.10	0.13	0.15	0.08
- Millet	-	1.03	0.64	0.03	-	0.33	-	-	0.40	-
- Millet/cowpea	0.44	-	0.35	0.53	-	-	-	-	-	0.12
- Groundnuts	0.13	0.07	0.08	0.06	0.55	0.13	0.36	0.55	0.10	0.35
Cultivated area (ha)	3.56	3.57	3.56	3.06	3.72	3.64	3.26	3.68	3.64	3.54
Area cultivated int. (ha)	2.96	3.57	2.64	1.83	3.72	2.87	2.01	3.68	2.72	2.12
Area cultivated ext. (ha)	0.60	-	0.92	1.12	-	0.77	1.01	-	0.91	0.88
Area abandoned (ha)	-	-	-	0.11	-	-	0.24	-	-	0.54
Production (kg)										
- cereals	1510	1118	1337	1352	1051	1453	1431	1090	1492	1483
- groundnuts	42	12	109	239	107	41	163	103	29	157
- cowpeas	52	14	22	27	40	60	66	48	68	78
Sales (kg)										
- cereals	96	122	37	48	133	44	48	147	122	150
- groundnuts	30	-	64	77	2	38	51	-	-	2
- cowpeas	44	-	12	14	-	38	50	-	55	64
Purchases (kg)										
- cereals	437	280	408	506	346	405	334	347	420	523
- groundnuts	-	31	-	-	-	-	-	-	-	-
Shortages										
- in 100 kilocalories	-	1621	176	-	1276	23	-	1184	-	-
- in proteins (1000 g)	-	17	-	-	9	-	-	7	-	-
Reserve Stock (kg) (cereals)	-	-	-	221	-	-	223	-	-	247
Degree of autoproduction <sup>4</sup>	72%	51%	68%	78%	56%	70%	78%	57%	71%	80%

(1) The objective is a summation of variables measured in kilocalories, proteins and kilograms. For this reason, the units of the objective cannot be presented.

(2) This refers only to the area cultivated in period 1 (the first-stage variables).

(3) This refers only to the area sown in stage 2.

(4) Ratio of the value of (all) agricultural production in kilocalories to the energy demand of the members of the Household.

**Table 3: Some results of the perfect information models and the model in which the first-stage strategies correspond to the average static model strategies**

	Perfect information models (15)									Static first-stage strategy		
	late			normal			early					
<i>Rainfall period 1</i>												
Value of the objective function <sup>1</sup> ( $\times 10^5$ )	11.3			8.15			7.26			10.27		
<i>Rainfall period 2</i>	bad	aver.	good	bad	aver.	good	bad	aver.	good	bad	aver.	good
Degree of autoproduction <sup>2</sup>	52%	68%	82%	57%	72%	84%	58%	75%	86%	53%	72%	80%

(1) The objective is a summation of variables measured in kilocalories, proteins and kilograms. For this reason, the units of the objective cannot be presented.

(2) Ratio of the value of (all) agricultural production in kilocalories to the energy demand of the members of the Household.

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