A JELS Stochastic inventory model with random demand

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Abstract

A stochastic joint lot size model has been developed in which demand of the customer and the stock level of the vendor are assumed to be identically distributed continuous random variables. The effective ways for a compromise between the vendor and the customer at a common lot size with certain amount of price adjustments are determined and the methodology is explained through a numerical example.

Key words: Inventory control programming, Stochastic models, Truncated normal distribution, Joint economic lot size.

1 Introduction

The well known EOQ model used so far, for the determination of order quantity in an inventory management system is considered from either the customer's point of view or from the vendor's point of view. Such models are applicable when both the parties operate independently. This strategy places one of the parties in a disadvantageous position. Such type of problems are usually solved in a Just -in-Time(JIT) system of production where the order quantity and production batch quantity are identical. However JIT concept can only be successful if both the customer and the vendor have high flexible management systems. In most of the cases, such ideal conditions do not exist. Hence an alternative solution to the above problem can be obtained through a mutual negotiation between the customer and the vendor on order quantity, lead time etc., in which both the parties jointly agree to a management schedule in stead of the weaker of the two adjusting his cost policy according to the convenience of the stronger one. Some authors like Goyal [2], Banerjee [1] have discussed about such negotiations through a new version of the EOQ model with joint replenishment. Banerjee [1] developed a deterministic JELS model in which the respective lot sizes of the customer and the vendor were assumed to be uniform over time and shortages were not allowed, while in real life situations, the demands are usually random variables and shortages are very common. In this paper, an attempt has been made to formulate a stochastic JELS model in which the demand of the customer and the stock level of the vendor are identically distributed independent random variables defined over $[0, \infty)$. To be more realistic, a situation where shortages are allowed has been considered. At the end, a suggestion for optimization of individual lot size by introducing unit price adjustment through a discount factor has been given. The model is verified through a numerical example where the random variables are normally distributed in truncated sense. Many researchers like Joglekar[3], Lee [4], Monahan [5] have discovered various methods of discount polices to satisfy the vendor. This paper deals with a discount policy which causes no loss to both the parties and both are getting some benefit.

2 Notations

Throughout this paper, the following notations and assumptions have been used.
z = Variable lot size ($Z = z$)for the fixed time interval t.
X = Random variable representing the demand of the customer with density $f_p(x)$ and $\int_0^\infty f_p(x)dx = 1$.
Y =Random variable representing the stock level of the vendor with density $f_v(y)$ and $\int_0^\infty f_v(y)dy = 1$.
C_{p1} = Holding cost of the customer per unit item per t time units.
C_{p2} = Shortage cost of the customer per unit item per t time units.
C_{v1} = Holding cost of the vendor per unit item per t time unit.
C_{v2} = Shortage cost of the vendor per unit item per t time unit.
z_p^* = Optimum ELS of customer.
z_v^* =Optimum ELS of vendor.
z^* = Joint optimum lot size (JELS).
$C_p(z)$ = Total expected relevant cost of the customer for demand z.
$C_v(z)$= Total expected relevant cost of the vendor for stock level z.
$C(z)$ = Joint expected total relevant cost for a common lot size z of both the parties .
There is no set up cost and lead time is zero.

3 Effect of Individual Optimization

The total relevant cost of the customer is

$$C_p(z) = C_{p1}\int_0^z (z-x)f_p(x)dx + C_{p2}\int_z^\infty (x-z)f_p(x)dx$$

$C_p(z)$ is a convex function of z and its global minimum occurs at z_p^* where

$$\phi_p(z_p^*) = \int_0^{z_p^*} f_p(x)dx = \frac{C_{p2}}{C_{p1}+C_{p2}}$$

The minimum cost of the customer is $C_p(z_p^*)$. Similarly the total relevant cost of the vendor is

$$C_v(z) = C_{v1}\int_0^z (z-y)f_v(y)dy + C_{v2}\int_z^\infty (y-z)f_v(y)dy$$

$C_v(z)$ is a convex function of z and its minimum occurs at z_v^* where

$$\phi_v(z_v^*) = \int_0^{z_v^*} f_v(y)dy = \frac{C_{v2}}{C_{v1}+C_{v2}}$$

The minimum cost of the vendor is $C_v(z_v^*)$.

If the customer's ELS (z_p^*) is adopted by both parties, the vendor's total relevant cost is $C_v(z_p^*)$. Since $C_v(z)$ is a convex function and z_v^* is its minimum point, so $C_v(z_p^*) > C_v(z_v^*)$, i.e the vendor will be in a disadvantageous position with a loss of amount

$$ACP_v(z_v^* \to z_p^*) = C_v(z_p^*) - C_v(z_v^*)$$

which is his absolute cost penalty. Similarly if the vendor's ELS (z_v^*) is adopted by both the parties, the customer's total relevant cost is $C_p(z_v^*)$. Since $C_p(z)$ is a convex function and z_p^* is its minimum point, so $C_p(z_v^*) > C_p(z_p^*)$ i.e the customer will be in a disadvantageous position with a loss of amount

$$ACP_p(z_p^* \to z_v^*) = C_p(z_v^*) - C_p(z_p^*)$$

which is his absolute cost penalty. Hence both the parties are in disadvantageous position if one adopts others ELS instead of his own. If the optimum lot sizes of the vendor and the customer are equal, then both the parties will automatically compromise at the same lot size. If the optimum lot sizes are different, then it is necessary to find out some intermediate common lot size for which one party will be gainer and the other will be loser.

Next section is devoted towards the determination of such a lot size which will benefit both the customer and the vendor without the need for cost sacrifice on the part of any. Using the nice properties of convex functions i.e $C(z)$, $C_p(z)$, $C_v(z)$, we shall prove that shifting from any initial status of the individual ELS to z^* is always workable and shifting from any initial status z_0 other than the individual ELS is workable in most of the situations. That is the gain of one party exceeds the loss of another party.

4 Formulation of Stochastic JELS Model

Once the customer and the vendor fail to convince each other to accept his ELS, they may come to a point of compromise in which the total cost of the inventory system is minimum in stead of individual cost being minimum. In this case it is said to be a Joint Economic Lot Size (JELS) model.

Suppose the demand of the customer and stock level of the vendor are identically distributed continuous random variables with common density function $f_p(x) = f_v(y) = f(s)$ say. Let (X, Y) be the joint continuous random variable representing the random demand of the customer and the random stock level maintained by the vendor, with $f(x, y)$ as the joint density function. Since X and Y are independent random variables so $f(x, y) = f_p(x).f_v(y)$. The expected joint relevant cost, $C(z)$, for any lot size z is the sum of the costs in all possible cases are calculated for t time period is,

$$\begin{aligned} C(z) &= \int_0^z \int_0^z (C_{p1}(z-x) + C_{v1}(z-y))f(x,y)dydx \\ &+ \int_0^z \int_z^\infty (C_{p1}(z-x) + C_{v2}(y-z))f(x,y)dydx \\ &+ \int_z^\infty \int_0^z (C_{p2}(x-z) + C_{v1}(z-y))f(x,y)dydx \\ &+ \int_z^\infty \int_z^\infty (C_{p2}(x-z) + C_{v2}(y-z))f(x,y)dydx \end{aligned}$$

$$\begin{aligned} &= \int_0^z C_{p1}(z-x)f_p(x)dx \int_0^z f_v(y)dy + \int_0^z f_p(x)dx \int_0^z C_{v1}(z-y)f_v(y)dy \\ &+ \int_0^z C_{p1}(z-x)f_p(x)dx \int_z^\infty f_v(y)dy + \int_0^z f_p(x)dx \int_z^\infty C_{v2}(y-z)f_v(y)dy \\ &+ \int_z^\infty C_{p2}(x-z)f_p(x)dx \int_0^z f_v(y)dy + \int_z^\infty f_p(x)dx \int_0^z C_{v1}(z-y)f_v(y)dy \\ &+ \int_z^\infty C_{p2}(x-z)f_p(x)dx \int_z^\infty f_v(y)dy + \int_z^\infty f_p(x)dx \int_z^\infty C_{v2}(y-z)f_v(y)dy \end{aligned}$$

$$\begin{aligned} &= \int_0^\infty f_v(y)dy \left[\int_0^z C_{p1}(z-x)f_p(x)dx + \int_z^\infty C_{p2}(x-z)f_p(x)dx\right] \\ &+ \int_0^\infty f_p(x)dx \left[\int_0^z C_{v1}(z-y)f_p(v)dy + \int_z^\infty C_{v2}(y-z)f_v(y)dy\right] \end{aligned}$$

Since $\int_0^\infty f_p(x)dx = \int_0^\infty f_v(y)dy = 1$ and X and Y are identically distributed random variables, so the above equation takes the form,

$$\begin{aligned} C(z) &= (C_{p1} + C_{v1}) \int_0^z (z-s)f(s)ds + (C_{p2} + C_{v2}) \int_z^\infty (s-z)f(s)ds \\ &= C_p(z) + C_v(z) \end{aligned}$$

$C(z)$, being the sum of two convex functions is a convex function of z and its minimum occurs at z^* where,

$$\phi(z^*) = \int_0^{z^*} f(s)ds = \frac{C_{p2} + C_{v2}}{C_{p1} + C_{p2} + C_{v1} + C_{v2}}$$

The minimum total joint cost of the system is $C(z^*)$. If the customer and the vendor accepts this joint lot size in place of their own then their expected cost will be $C_p(z^*)$ and $C_v(z^*)$ respectively. To express the complicated expressions appearing in the subsequent discussions in simple forms, we define

$$\alpha = \frac{C_{p2}}{C_{v2}}$$

$$\beta = \frac{C_{p1} + C_{p2}}{C_{v1} + C_{v2}}$$

α is the ratio of the shortage cost of the customer to that of vendor and β is the ratio of the total cost of the customer to that of the vendor.

The following relations can be easily derived from the above equations.

$$\phi(z^*) = \left[\frac{1 + \frac{1}{\alpha}}{1 + \frac{1}{\beta}}\right] \phi_p(z_p^*) = \left[\frac{1+\alpha}{1+\beta}\right] \phi_v(z_v^*)$$

LEMMA 4.1 *If* $\alpha < \beta$ *then* $z_p^* < z^* < z_v^*$
If $\alpha = \beta$ *then* $z_p^* = z^* = z_v^*$
If $\alpha > \beta$ *then* $z_p^* > z^* > z_v^*$

Proof of the lemma is easy and straight forward since ϕ, ϕ_p, ϕ_v are bijective and monotonically increasing functions.The physical interpretation of the above lemma is that,the joint lot size of the inventory system always lies in between the individual optimum lot sizes when both the ratios are different.

5 Effect of individual optimization on adopting JELS

Since the joint lot size of the system minimizes the total cost of the system as well as lies in between the individual optimum lot sizes, both the parties have an option to accept this joint lot size. If $\alpha = \beta$ then z_p^*, z^* and z_v^* will coincide. Hence in this case there will be no necessity of negotiation between two parties. Now we discuss different cases related to $\alpha \neq \beta$.

CASE-I:(Utility of JELS if customer's ELS is in effect)

Suppose the customer's ELS, z_p^* is in effect and each of the vendor and the customer change their lot size to z^*.

(i) If $\alpha < \beta$ then $z_p^* < z^* < z_v^*$. Since $C_p(z)$ and $C_v(z)$ are convex functions of z with minimum at z_p^* and z_v^* respectively. So $C_p(z^*) > C_p(z_p^*)$ and $C_v(z_p^*) > C_v(z^*)$. Hence if the vendor accepts the JELS instead of customer's ELS , the absolute cost advantage of the vendor i.e $ACA_v(z_p^* \to z^*)$ is

$$ACA_v(z_p^* \to z^*) = C_v(z_p^*) - C_v(z^*)$$

The absolute cost penalty of the customer, if he accepts z^* in stead of his own lot size z_p^* is

$$ACP_p(z_p^* \to z^*) = C_p(z^*) - C_p(z_p^*)$$

$$\begin{aligned} ACA_v(z_p^* \to z^*) - ACP_p(z_p^* \to z^*) &= [C_v(z_p^*) - C_v(z^*)] - [C_p(z^*) - C_p(z_p^*)] \\ &= [C_v(z_p^*) + C_p(z_p^*)] - [C_p(z^*) + C_v(z^*)] \\ &= C(z_p^*) - C(z^*) \end{aligned}$$

which is positive since $C(z)$ is a convex function. That is, absolute cost advantage of the vendor is more than the absolute cost penalty of the customer. The joint cost advantage is defined as

$$\begin{aligned} JACA_v(z_p^* \to z^*) &= ACA_v(z_p^* \to z^*) - ACP_p(z_p^* \to z^*) \\ &= C(z_p^*) - C(z^*) \end{aligned}$$

(ii) If $\alpha > \beta$ then $z_p^* > z^* > z_v^*$. As in (i), it is trivial that $C_v(z_p^*) > C_v(z^*)$ and $C_p(z^*) > C_p(z_p^*)$. Then the vendor gets some benefit if the customer's ELS, (z_p^*), is in effect and he changes it to JELS. Hence his absolute cost advantage is $ACA_v(z_p^* \to z^*)$ and the customer faces a loss of amount $ACP_p(z_p^* \to z^*)$. As in (I) it is easy to see that the vendor and the customer get a joint absolute cost advantage $JACA_v(z_p^* \to z^*) = C(z_p^*) - C(z^*)$.

Hence in case $\alpha \neq \beta$, the above discussion implies that the vendor is in advantageous position if the customer's ELS is in effect and both parties change it to the JELS. So to continue the inventory management for a long period and to make the negotiation successful, the vendor can compromise the loss of the customer by fixing some price discount. To attract the customer to change his own lot size to joint lot size the vendor may offer a total price discount d for which the upper and lower bounds are given by

$$ACP_p(z_p^* \to z^*) \leq d \leq ACA_v(z_p^* \to z^*)$$

If $d = ACP_p(z_p^* \to z^*)$ then all benefits, by adopting JELS will go to the vendor and the customer is indifferent between his own ELS and JELS. On the other hand if $d = ACA_v(z_p^* \to z^*)$ then all benefits will go to the customer and the vendor's total relevant cost remains unaltered. To be fair to both, the joint benefit for total demand may be divided equally between both parties. Hence the actual discount given by the vendor is

$$\frac{JACA_v(z_p^* \to z^*)}{2}$$

and the optimum unit price discount $\triangle$ is

$$\triangle = \frac{JACA_v(z_p^* \to z^*)}{2m}$$

Where m is the expectation of the random variable X(or Y since they are identical)

CASE II (Utility of JELS when vendor's ELS is in effect)

Suppose the vendor's ELS is in effect and both the customer and the vendor change their individual lot size to JELS. As in case I, we can analyze the effectiveness of the JELS on each party for different values of α and β.

(i) If $\alpha < \beta$ then $z_p^* < z^* < z_v^*$ The absolute cost advantage of the customer is,

$$ACA_p(z_v^* \to z^*) = C_p(z_v^*) - C_p(z^*)$$

which is always positive and the absolute cost penalty of the vendor is

$$ACP_v(z_v^* \to z^*) = C_v(z^*) - C_v(z_v^*)$$

As discussed in case I, the joint absolute cost advantage can be

$$\begin{aligned} JACA_p(z_v^* \to z^*) &= ACA_p(z_v^* \to z^*) - ACP_v(z_v^* \to z^*) \\ &= C(z_v^*) - C(z^*) \end{aligned}$$

which is positive since $C(z)$ is a convex function and z^* is its minimum point.Thus the customer gains and the vendor loses in this situation.

(ii) If $\alpha > \beta$ then $z_p^* > z^* > z_v^*$

Then the customer gets some benefit if the vendor's ELS, z_v^* is in effect and he changes it to JELS. Hence his absolute cost advantage is $ACA_p(z_v^* \to z^*)$ and the vendor faces a loss of amount $ACP_v(z_v^* \to z^*)$. Both parties get a joint absolute cost advantage $JACA_p(z_v^* \to z^*)$.

Hence in case $\alpha \neq \beta$, the above discussion implies that the customer is in an advantageous position if the vendor's ELS is in effect and both parties change it to the JELS. In order to continue his purchase from the vendor for a long period, the customer can persuade the vendor to change his lot size to JELS by offering some price increase, d' for which the upper and the lower bounds are given by

$$ACP_v(z_v^* \to z^*) \leq d' \leq ACA_p(z_v^* \to z^*)$$

If $d' = ACP_v(z_v^* \to z^*)$, all benefits by adopting JELS, will go to the customer where if $d' = ACA_p(z_v^* \to z^*)$, all benefits will go to the vendor. Hence to be fair to both the joint benefit may be divided equally between the vendor and the customer. The actual value of d' is,

$$\frac{JACA_p(z_v^* \to z^*)}{2}$$

and the optimum unit price discount i.e ∇ is,

$$\nabla = \frac{JACA_p(z_v^* \to z^*)}{2m}$$

CASE III(Utility of JELS when any ELS is in effect)

In this case the effect of JELS is compared with any ELS z_0 other than individual ELS i.e z_p^* and z_v^*.

(i) If $C_v(z^*) > C_v(z_0)$ and $C_p(z^*) < C_p(z_0)$ then vender will be in disadvantageous position and the customer is in advantageous position. $ACP_v(z_0 \to z^*)$ and $ACA_p(z_0 \to z^*)$ are the vendor's absolute cost penalty and customer's absolute cost advantage respectively. Hence the joint absolute cost advantage is

$$JACA_p(z_0 \to z^*) = ACA_p(z_0 \to z^*) - ACP_v(z_0 \to z^*)$$

The unit price discount can be determined as in case II.

(ii) If $C_v(z^*) < C_v(z_0)$ and $C_p(z^*) < C_p(z_0)$ then both the vender the customer will be advantageous position with absolute cost advantage $ACA_v(z_0 \rightarrow z^*)$ and $ACA_p(z_0 \rightarrow z^*)$. Hence in this case it is more advisable to accept the JELS in place of z_0.

(iii) If $C_v(z^*) > C_v(z_0)$ and $C_p(z^*) > C_p(z_0)$ then both the vender and the customer will be in disadvantageous position and their absolute cost penalty are respectively $ACP_v(z_0 \rightarrow z^*)$ and $ACP_p(z_0 \rightarrow z^*)$. Hence in this case it is more advisable to accept z_0 in place of the JELS.

(iv) If $C_v(z^*) < C_v(z_0)$ and $C_p(z^*) > C_p(z_0)$ then vender will be in advantageous position and the customer will be in disadvantageous position. $ACA_v(z_0 \rightarrow z^*)$ and $ACP_p(z_0 \rightarrow z^*)$ are the vendor's absolute cost advantage and customer's absolute cost penalty respectively. Hence the joint absolute cost advantage is

$$JACA_v(z_0 \rightarrow z^*) = ACA_v(z_0 \rightarrow z^*) - ACP_p(z_0 \rightarrow z^*)$$

The unit price discount can be determined as in case I.

6 Numerical example

Suppose an inventory item is produced by a vendor on a lot for lot basis by the order of the customer. The vendor is the customer's sole source for this item. A single customer weekly orders and buys a batch of this item from the vendor. Suppose the demand and the stock size are independent and identically distributed random variables belonging to the standard normal distribution. Since these random variables are distributed over $[0, \infty)$, so we may consider them as normally distributed random variables truncated at 0 i.e the density function, $f(s)$ is

$$f(s) = \begin{cases} (2/\sqrt{2\pi})e^{-\frac{s^2}{2}} & s \geq 0 \\ 0 & s < 0 \end{cases}$$

$C_{p1} = \$80$, $C_{p2} = \$20$, $C_{v1} = \$120$, $C_{v2} = \$40$. The costs related to the inventory system are expressed in terms of 100 dollars. There is no set up cost and lead time is zero.
Here $m = \sqrt{\frac{2}{\pi}}$, $\alpha = 0.5$, $\beta = 0.625$, $z^* = 0.29$, $z_p^* = 0.25$, $z_v^* = 0.32$.

$$C_v(z^*) = 2\left[\frac{z^*}{2}\frac{\alpha-\beta}{\beta+1}C_{v2} + \frac{C_{v1}+C_{v2}}{\sqrt{2\pi}}e^{-\frac{(z^*)^2}{2}} - \frac{C_{v1}}{\sqrt{2\pi}}\right] = \$2576.62$$

$$C_p(z^*) = 2\left[\frac{z^*}{2}\frac{\beta-\alpha}{\beta+1}C_{v2} + \frac{C_{p1}+C_{p2}}{\sqrt{2\pi}}e^{-\frac{(z^*)^2}{2}} - \frac{C_{p1}}{\sqrt{2\pi}}\right] = \$1356.44$$

$$C_v(z_v^*) = 2\left[\frac{C_{v1}+C_{v2}}{\sqrt{2\pi}}e^{-\frac{(z_v^*)^2}{2}} - \frac{C_{v1}}{\sqrt{2\pi}}\right] = \$2554.36$$

$$C_p(z_v^*) = 2\left[\frac{z_v^* C_{v2}(\beta-\alpha)}{2} + \frac{\beta(C_{v1}+C_{v2})e^{-\frac{(z_v^*)^2}{2}}}{\sqrt{2\pi}} - \frac{C_{p1}}{\sqrt{2\pi}}\right] = \$1357.53$$

$$C_v(z_p^*) = 2\left[\frac{z_p^* C_{p2}}{2}\left(\frac{1}{\beta} - \frac{1}{\alpha}\right) + \frac{C_{v1}+C_{v2}}{\sqrt{2\pi}}e^{-\frac{(z_p^*)^2}{2}} - \frac{C_{v1}}{\sqrt{2\pi}}\right] = \$2598.76$$

$$C_p(z_p^*) = 2\left[\frac{C_{p1}+C_{p2}}{\sqrt{2\pi}}e^{-\frac{(z_p^*)^2}{2}} - \frac{C_{p1}}{\sqrt{2\pi}}\right] = \$1350.284$$

$ACP_p(z_p^* \rightarrow z_v^*) = \$7.2478$, $ACP_v(z_v^* \rightarrow z_p^*) = \$44.4032$. Thus either in case $z_p^* \rightarrow z_v^*$ or $z_v^* \rightarrow z_p^*$, the customer or the vendor will be in loss. Hence they will search for a compromise at JELS. Suppose the customer's ELS is in effect. Then $ACA_v(z_p^* \rightarrow z^*) = \$22.1442$ and $ACP_p(z_p^* \rightarrow z^*) = \$6.1597$. Hence discount given by the vendor to the customer per unit item over total expected lot size of the customer is \$10.01680. If the customer is persuaded to change his lot size from 0.25 to 0.29 then the vendor has to offer a total discount of \$7.99225. Vendor's net gain is \$14.15195. Hence customer's gain per unit item is

$$(1/m)\left[(JACA_v(z_p^* \rightarrow z^*)/2) - ACP_p(z_p^* \rightarrow z^*)\right] = \$2.29676$$

Therefore the total cost of the customer per unit item is reduced to $(C_{p1} + C_{p2}) - \triangle = \$9989.98$ in stead of the original cost i.e $C_{p1} + C_{p2} = \$10,000$. In the given example, the lot size and the absolute cost advantages/penalties as well as the net gains/penalties and unit discounts in terms of rupees appear to be insignificant. But in case of heavy industry where continuous production is made in units of metric tons and the money transactions are made in terms of crores of rupees (like iron and steel industries or sugar producing industries etc.), the above results are of immense significance.

7 Conclusion

In this paper a detailed analysis has been made to show how inventory related costs vary through closer interaction between the vendor and the customer. The unit price and the order quantity etc are settled by negotiation between both the parties to minimize the total relevant costs. If JELS is adopted by both, the gain or loss are to be shared reasonably between them so that both will come to a mutual compromise. JELS model not only minimizes the total relevant cost of the system but also searches a common lot size with no loss to both. In this model the set up cost is assumed to be zero. Even if we consider the set up cost of both the parties, this will not affect the optimum lot sizes and the discount factor. It may be noted that the effect of this JELS model can be verified in various other situations with demand satisfying different continuous probability distributions like exponential, gamma, chi-square, log normal etc. How ever the demand of the customer and the stock level of the vendor are non-negative quantities. In practical situations, the negative lot size bears no meaning. Hence If X is distributed over $(-\infty, \infty)$, then it should be considered with conditional probability 0 for x negative. Following are the further research scope of this paper.

(i) In many real life problems the random variables X and Y may not be identically distributed random variables. The joint cost equation can be determined if X and Y follow any general joint distribution and the model can be studied in that situation.

(ii) The effect of JELS may be studied in more practical situations where non zero lead time, multiple price break, order quantity etc are involved.

(iii) The idea of negotiation between single customer and single vendor may extended to different situations involving single customer multi vendor, multi customer multi vendor and multi customer single vendor.

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